

Half-Step Continuous Block Method for the Solutions of Modeled Problems of Ordinary Differential Equations

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ABSTRACT

In this paper, we developed a new continuous block method using the approach of collocation of the differential system and interpolation of the power series approximate solution. A constant step length within a half step interval of integration was adopted. We evaluated at grid and off grid points to get a continuous linear multistep method. The continuous linear multistep method is solved for the independent solution to yield a continuous block method which is evaluated at selected points to yield a discrete block method. The basic properties of the block method were investigated and found to be consistent and zero stable hence convergent. The new method was tested on real life problems namely: SIR model, Growth model and Mixture Model. The results were found to compete favorably with the existing methods in terms of accuracy and error bound.

Keywords: Approximate Solution; Interpolation; Collocation; Half Step; Converges; Block Method

1. Introduction

We consider the numerical solution of first order initial value problems of the form:

$$y' = f(x, y), y(x_0) = y_0 \quad (1)$$

where f is continuous and satisfies Lipchitz's condition that guarantees the uniqueness and existence of a solution.

Problem in the form (1) has wide application in physical science, engineering, economics, etc. Very often, these problems do not have an analytical solution, and this has necessitated the deviation of numerical schemes to approximate their solutions.

In the past, scholars have developed a continuous linear multistep in solving (1). These authors proposed methods with different basis functions and among them were [1-6] to mention a few.

These authors proposed methods ranging from predictor corrector method to discrete block method.

Scholars later proposed block method. This block method has the properties of Runge-kutta method for being self-starting and does not require development of separate predictors or starting values. Among these au-

thors are [7-12]. Block method was found to be cost effective and gave better approximation.

This paper is divided into sections as follows: Section 1 is the introduction and background of the study; Section 2 contains the discussion about the methodology involved in deriving the continuous multistep method and the continuous block method. Section 3 considers the analysis of the block method in terms of the order, zero stability and the region of absolute stability. Section 4 focuses on the application of the new method on some numeric examples and Section 5 is on the discussion of result. We tested our method on first order ordinary differential equations and compared our result with existing methods.

2. Methodology

Consider power series approximate solution in the form

$$y(x) = \sum_{j=0}^{s+r-1} a_j x^j \quad (2)$$

where S and r are the number of interpolation and collocation points respectively.

The first derivative of (2) gives

$$y'(x) = \sum_{j=0}^{s+r-1} ja_j x^{j-1} \tag{3}$$

Substituting (3) into (2) gives

$$f(x, y) = \sum_{j=0}^{s+r-1} ja_j x^{j-1} \tag{4}$$

Collocating (3) at $x_{n+s}, s = 0\left(\frac{1}{12}\right)\frac{1}{2}$ and interpolating

(2) at x_n gives a system of non-linear equation in the form

$$AX = U \tag{5}$$

where

$$A = \begin{bmatrix} 1 & x_n & x_n^2 & x_n^3 & x_n^4 & x_n^5 & x_n^6 & x_n^7 \\ 0 & 1 & 2x_n & 3x_n^2 & 4x_n^3 & 5x_n^4 & 6x_n^5 & 7x_n^6 \\ 0 & 1 & 2x_{n+\frac{1}{12}} & 3x_{n+\frac{1}{12}}^2 & 4x_{n+\frac{1}{12}}^3 & 5x_{n+\frac{1}{12}}^4 & 6x_{n+\frac{1}{12}}^5 & 7x_{n+\frac{1}{12}}^6 \\ 0 & 1 & 2x_{n+\frac{1}{6}} & 3x_{n+\frac{1}{6}}^2 & 4x_{n+\frac{1}{6}}^3 & 5x_{n+\frac{1}{6}}^4 & 6x_{n+\frac{1}{6}}^5 & 7x_{n+\frac{1}{6}}^6 \\ 0 & 1 & 2x_{n+\frac{1}{4}} & 3x_{n+\frac{1}{4}}^2 & 4x_{n+\frac{1}{4}}^3 & 5x_{n+\frac{1}{4}}^4 & 6x_{n+\frac{1}{4}}^5 & 7x_{n+\frac{1}{4}}^6 \\ 0 & 1 & 2x_{n+\frac{1}{3}} & 3x_{n+\frac{1}{3}}^2 & 4x_{n+\frac{1}{3}}^3 & 5x_{n+\frac{1}{3}}^4 & 6x_{n+\frac{1}{3}}^5 & 7x_{n+\frac{1}{3}}^6 \\ 0 & 1 & 2x_{n+\frac{5}{12}} & 3x_{n+\frac{5}{12}}^2 & 4x_{n+\frac{5}{12}}^3 & 5x_{n+\frac{5}{12}}^4 & 6x_{n+\frac{5}{12}}^5 & 7x_{n+\frac{5}{12}}^6 \\ 0 & 1 & 2x_{n+\frac{1}{2}} & 3x_{n+\frac{1}{2}}^2 & 4x_{n+\frac{1}{2}}^3 & 5x_{n+\frac{1}{2}}^4 & 6x_{n+\frac{1}{2}}^5 & 7x_{n+\frac{1}{2}}^6 \end{bmatrix}, U = \begin{bmatrix} y_n \\ f_n \\ f_{n+\frac{1}{12}} \\ f_{n+\frac{1}{6}} \\ f_{n+\frac{1}{4}} \\ f_{n+\frac{1}{3}} \\ f_{n+\frac{5}{12}} \\ f_{n+\frac{1}{2}} \end{bmatrix}$$

Solving (5) for the a_j s and substituting back into (4) gives a continuous multistep method in the form

where $a_0 = 1$ and the coefficients of f_{n+j} gives

$$y(x) = \alpha_0 y_n + h \sum_{j=0}^{\frac{1}{2}} \beta_j(x) f_{n+j}, j = 0\left(\frac{1}{12}\right)\frac{1}{2} \tag{6}$$

$$\beta_0 = \left(\frac{1}{120}\right)(124416t^7 - 254016t^6 + 211680t^5 - 92610t^4 + 22736t^3 - 3087t^2 + 210t)$$

$$\beta_{\frac{1}{12}} = \left(-\frac{1}{35}\right)(-124416t^7 - 241920t^6 - 187488t^5 + 73080t^4 - 14616t^3 + 1260t^2)$$

$$\beta_{\frac{1}{6}} = \left(\frac{1}{35}\right)(311040t^7 - 57456t^6 + 414288t^5 - 145215t^4 + 24570t^3 - 1575t^2)$$

$$\beta_{\frac{1}{4}} = \left(-\frac{1}{105}\right)(-1244160t^7 - 2177280t^6 - 1463616t^5 + 468720t^4 - 71120t^3 + 4200t^2)$$

$$\beta_{\frac{1}{3}} = \left(\frac{1}{70}\right)(622080t^7 - 1028160t^6 + 647136t^5 - 193410t^4 + 27720t^3 - 1575t^2)$$

$$\beta_{\frac{5}{12}} = \left(-\frac{1}{35}\right)(-124416t^7 + 193536t^6 - 114912t^5 + 32760t^4 - 4536t^3 + 252t^2)$$

$$\beta_{\frac{1}{2}} = \left(\frac{1}{105}\right)(62208t^7 - 90720t^6 + 51408t^5 - 14175t^4 + 1918t^3 - 105t^2)$$

where $t = \frac{x-x_n}{h}$. Solving (6) for the independent solution gives a continuous block method in the form

$$y_{n+k} = \sum_{j=0}^{\mu-1} \frac{(jh)^m}{m!} y_n^{(m)} + h^\mu \sum_{j=0}^s \sigma_j(x) f_{n+j} \quad (7)$$

where μ is the order of the differential equation s is the collocation points. Hence the coefficient of f_{n+j} in (7)

$$\begin{aligned} \sigma_0 &= \left(\frac{1}{120}\right)(124416t^7 - 254016t^6 + 211680t^5 \\ &\quad - 92610t^4 + 22736t^3 - 3087t^2 + 210t) \\ \sigma_{\frac{1}{12}} &= \left(-\frac{1}{35}\right)(-124416t^7 - 241920t^6 - 187488t^5 \\ &\quad + 73080t^4 - 14616t^3 + 1260t^2) \\ \sigma_{\frac{1}{6}} &= \left(\frac{1}{35}\right)(311040t^7 - 57456t^6 + 414288t^5 \\ &\quad - 145215t^4 + 24570t^3 - 1575t^2) \\ \sigma_{\frac{1}{4}} &= \left(-\frac{1}{105}\right)(-1244160t^7 - 2177280t^6 - 1463616t^5 \\ &\quad + 468720t^4 - 71120t^3 + 4200t^2) \\ \sigma_{\frac{1}{3}} &= \left(\frac{1}{70}\right)(622080t^7 - 1028160t^6 + 647136t^5 \\ &\quad - 193410t^4 + 27720t^3 - 1575t^2) \\ \sigma_{\frac{5}{12}} &= \left(-\frac{1}{35}\right)(-124416t^7 + 193536t^6 - 114912t^5 \\ &\quad + 32760t^4 - 4536t^3 + 252t^2) \end{aligned}$$

$$b = \begin{bmatrix} \frac{2713}{30240} & \frac{-15487}{241920} & \frac{293}{5670} & \frac{-6737}{241920} & \frac{263}{30240} & \frac{-863}{725760} \\ \frac{47}{378} & \frac{11}{15120} & \frac{83}{2835} & \frac{-269}{15120} & \frac{11}{1890} & \frac{-37}{45360} \\ \frac{27}{224} & \frac{387}{8960} & \frac{17}{210} & \frac{-243}{8960} & \frac{9}{1120} & \frac{-29}{26880} \\ \frac{116}{945} & \frac{32}{945} & \frac{376}{2835} & \frac{29}{1890} & \frac{4}{945} & \frac{-2}{2835} \\ \frac{725}{6048} & \frac{2125}{48384} & \frac{125}{1134} & \frac{3875}{48384} & \frac{235}{6048} & \frac{-275}{145152} \\ \frac{9}{70} & \frac{9}{560} & \frac{17}{105} & \frac{9}{560} & \frac{9}{70} & \frac{41}{1680} \end{bmatrix}$$

$$\sigma_{\frac{1}{2}} = \left(\frac{1}{105}\right)(62208t^7 - 90720t^6 + 51408t^5 \\ - 14175t^4 + 1918t^3 - 105t^2)$$

where $t = \frac{x-x_n}{h}$.

Evaluating (2.5) at $t = \frac{1}{12}\left(\frac{1}{12}\right)\frac{1}{2}$ gives a discrete block formula of the form

$$Y_m = ey_n + hdf(y_n) + hdf(Y_m) \quad (8)$$

where e, d , are $r \times r$ matrix

where

$$d = \begin{bmatrix} \frac{19087}{725760} & \frac{1139}{45360} & \frac{137}{5376} & \frac{143}{5670} & \frac{3715}{145152} & \frac{41}{1680} \end{bmatrix}^T$$

$$Y_m = \left[y_{n+\frac{1}{12}}, y_{n+\frac{1}{6}}, y_{n+\frac{1}{4}}, y_{n+\frac{1}{3}}, y_{n+\frac{5}{12}}, y_{n+\frac{1}{2}} \right]^T$$

$$e = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

3. Analysis of the Basic Properties of Our New Method

3.1. Order of the Method

Let the linear operator $L\{y(x); h\}$ associated with the block formular be defined as

$$L\{y(x); h\} = A^{(0)}Y_m - ey_n - h^\mu df(y_n) - h^\mu bF(Y_m) \tag{9}$$

expanding in Taylor series and comparing the coefficient of h gives

$$L\{y(x); h\} = c_0 y(x) + c_1 h y^{(x)} + c_1 h y^{(x)} + \dots + c_p h^p y^{(p)}(x) + c_{p+1} h^{p+1} y^{(p+1)}(x) + c_{p+2} h^{p+2} y^{(p+2)}(x) \tag{10}$$

Definition:-The linear operator L and the associated continuous linear multistep method (3.1) are said to be of order p if $c_0 = c_1 = c_2 = \dots = c_p = 0$ and $c_{p+1} \neq 0$ is

called the error constant and implies that the local truncation error is given by

$$t_{n+k} = C_{p+2} h^{p+1} y^{(p+1)}(x_n) + O(h^{p+2}).$$

For our method;

$$L\{y(x); h\} = \begin{bmatrix} y_{n+\frac{1}{12}} \\ y_{n+\frac{1}{6}} \\ y_{n+\frac{1}{4}} \\ y_{n+\frac{1}{3}} \\ y_{n+\frac{5}{12}} \\ y_{n+\frac{1}{2}} \end{bmatrix} - \begin{bmatrix} 1 & \frac{1}{12}h \\ 1 & \frac{1}{6}h \\ 1 & \frac{1}{4}h \\ 1 & \frac{1}{3}h \\ 1 & \frac{5}{12}h \\ 1 & \frac{1}{2}h \end{bmatrix} [y_n] - \begin{bmatrix} f_n \\ f_{n+\frac{1}{12}} \\ f_{n+\frac{1}{6}} \\ f_{n+\frac{1}{4}} \\ f_{n+\frac{1}{3}} \\ f_{n+\frac{5}{12}} \\ f_{n+\frac{1}{2}} \end{bmatrix} = 0$$

Expanding in Taylor series expansion gives

$$\begin{aligned} & \sum_{j=0}^{\infty} \frac{\left(\frac{1}{12}\right)^j}{j!} y_n^j - y_n - \frac{19087}{725760} h y_n' - \sum_{j=0}^{\infty} \frac{h^{j+1}}{j!} y_n^{j+1} \left\{ \frac{2713}{30240} \left(\frac{1}{12}\right)^j + \frac{15487}{241920} \left(\frac{1}{6}\right)^j \right. \\ & \left. - \frac{293}{5670} \left(\frac{1}{4}\right)^j + \frac{6737}{241920} \left(\frac{1}{3}\right)^j + \frac{263}{30240} \left(\frac{5}{6}\right)^j - \frac{863}{725760} \left(\frac{1}{2}\right)^j \right\} \\ & \sum_{j=0}^{\infty} \frac{\left(\frac{1}{6}\right)^j}{j!} y_n^j - y_n - \frac{1139}{45360} h y_n' - \sum_{j=0}^{\infty} \frac{h^{j+1}}{j!} y_n^{j+1} \left\{ \frac{47}{378} \left(\frac{1}{12}\right)^j + \frac{11}{15120} \left(\frac{1}{6}\right)^j \right. \\ & \left. + \frac{83}{2835} \left(\frac{1}{4}\right)^j - \frac{269}{15120} \left(\frac{1}{3}\right)^j - \frac{11}{1890} \left(\frac{5}{12}\right)^j - \frac{37}{45360} \left(\frac{1}{2}\right)^j \right\} \\ & \sum_{j=0}^{\infty} \frac{\left(\frac{1}{4}\right)^j}{j!} y_n^j - y_n - \frac{137}{5376} h y_n' - \sum_{j=0}^{\infty} \frac{h^{j+1}}{j!} y_n^{j+1} \left\{ \frac{27}{224} \left(\frac{1}{12}\right)^j + \frac{387}{8960} \left(\frac{1}{6}\right)^j \right. \\ & \left. + \frac{17}{210} \left(\frac{1}{4}\right)^j - \frac{243}{8960} \left(\frac{1}{3}\right)^j + \frac{9}{1120} \left(\frac{5}{12}\right)^j - \frac{29}{26880} \left(\frac{1}{2}\right)^j \right\} \\ & \sum_{j=0}^{\infty} \frac{\left(\frac{1}{3}\right)^j}{j!} y_n^j - y_n - \frac{143}{5670} h y_n' - \sum_{j=0}^{\infty} \frac{h^{j+1}}{j!} y_n^{j+1} \left\{ \frac{116}{945} \left(\frac{1}{12}\right)^j + \frac{32}{945} \left(\frac{1}{6}\right)^j \right. \\ & \left. + \frac{376}{2835} \left(\frac{1}{4}\right)^j + \frac{29}{1890} \left(\frac{1}{3}\right)^j + \frac{4}{945} \left(\frac{5}{12}\right)^j - \frac{2}{2835} \left(\frac{1}{2}\right)^j \right\} \end{aligned}$$

$$\sum_{j=0}^{\infty} \frac{\left(\frac{5}{12}\right)^j}{j!} y_n^j - y_n - \frac{3715}{145152} h y_n' - \sum_{j=0}^{\infty} \frac{h^{j+1}}{j!} y_n^{j+1} \left\{ \frac{725}{6048} \left(\frac{1}{12}\right)^j + \frac{2125}{48384} \left(\frac{1}{6}\right)^j + \frac{125}{1134} \left(\frac{1}{4}\right)^j + \frac{3875}{48384} \left(\frac{1}{3}\right)^j + \frac{235}{6048} \left(\frac{5}{12}\right)^j - \frac{275}{145152} \left(\frac{1}{2}\right)^j \right\}$$

$$\sum_{j=0}^{\infty} \frac{\left(\frac{1}{2}\right)^j}{j!} y_n^j - y_n - \frac{41}{1680} h y_n' - \sum_{j=0}^{\infty} \frac{h^{j+1}}{j!} y_n^{j+1} \left\{ \frac{9}{70} \left(\frac{1}{12}\right)^j + \frac{9}{560} \left(\frac{1}{6}\right)^j + \frac{17}{105} \left(\frac{1}{4}\right)^j + \frac{9}{560} \left(\frac{1}{3}\right)^j + \frac{9}{70} \left(\frac{5}{12}\right)^j - \frac{41}{1680} \left(\frac{1}{2}\right)^j \right\}$$

Equating coefficients of the Taylor series expansion to zero yield

$$c_0 = c_1 = \dots = c_6 = 0$$

Hence we arrived at a uniform order 6 for our method with error constants

$$c_7 = [3.78(-09)2.53(-09)3.49(-09)1.85(-09)1.68(-08)5.56(-08)]$$

3.2. Zero Stability

Definition: The block (8) is said to be zero stable, if the roots $Z_s, s = 1, 2, \dots, N$ of the characteristic polynomial $\rho(z)$ defined by $\rho(z) \det(zA^{(0)} - E)$ satisfies $|z_s| \leq 1$ and every root satisfying $|z_s| = 1$ have multiplicity not

exceeding the order of the differential equation. Moreover as $h \rightarrow 0, \rho(z) = z^{r-\mu} (z-1)^\mu$ where μ is the order of the differential equation, r is the order of the matrix $A^{(0)}$ and E .

For our method

$$\rho(z) = z \begin{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{bmatrix} = 0$$

$\rho(z) = z^5(z-1)$. Hence our method is zero stable.

urrence equation

$$y^{i+1} = M(Z) y^i, Z = \lambda h, i = 1, 2, \dots, \mu - 1$$

3.3 Region of Absolute Stability

The block formulated as a general linear method where it is partition in the form

$$\begin{bmatrix} Y \\ Y_{n-i} \end{bmatrix} = \begin{bmatrix} A_1 & B_1 [hf(y)] \\ A_2 & B_2 [y_n] \end{bmatrix}$$

The elements of A_1 and A_2 are obtained from the coefficients of the collocation points, B_1 and B_2 are obtained from the interpolation points.

Applying the test equation $y' = \lambda y$ leads to the re-

The stability function is given by

$$M(Z) = B_2 + ZA_2(I - ZA_1)^{-B_1}$$

and the stability polynomial of the method is given as

$$\rho(\lambda, Z) = \det(\lambda I - M(Z))$$

The region of absolute stability of the method is defined as $\rho(\lambda, Z) = 1, |\lambda| \leq 1$.

For our method, writing the block in partition form gives

$$\begin{bmatrix} y_n y_{n+\frac{1}{12}} y_{n+\frac{1}{6}} y_{n+\frac{1}{4}} y_{n+\frac{1}{3}} y_{n+\frac{5}{12}} y_{n+1} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 1 \\ \frac{19087}{725760} & \frac{2713}{30240} & \frac{-15487}{241920} & \frac{293}{5670} & \frac{-6737}{241920} & \frac{263}{30240} & \frac{-863}{725760} & \dots & 0 & 1 \\ \frac{1139}{45360} & \frac{47}{378} & \frac{11}{15120} & \frac{83}{2835} & \frac{-269}{15120} & \frac{11}{1890} & \frac{-37}{45360} & \dots & 0 & 1 \\ \frac{137}{5376} & \frac{27}{224} & \frac{387}{8960} & \frac{17}{210} & \frac{-243}{8960} & \frac{9}{1120} & \frac{-29}{26880} & \dots & 0 & 1 \\ \frac{143}{5670} & \frac{116}{945} & \frac{32}{945} & \frac{376}{2835} & \frac{29}{1890} & \frac{4}{945} & \frac{-2}{2835} & \dots & 0 & 1 \\ \frac{3715}{145152} & \frac{725}{6048} & \frac{2125}{48384} & \frac{125}{1134} & \frac{3875}{48384} & \frac{235}{6048} & \frac{-275}{145152} & \dots & 0 & 1 \\ \frac{41}{1680} & \frac{9}{70} & \frac{9}{560} & \frac{17}{105} & \frac{9}{560} & \frac{9}{70} & \frac{41}{1680} & \dots & 0 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \frac{19087}{725760} & \frac{2713}{30240} & \frac{-15487}{241920} & \frac{293}{5670} & \frac{-6737}{241920} & \frac{263}{30240} & \frac{-863}{725760} & \dots & 0 & 1 \\ \frac{41}{1680} & \frac{9}{70} & \frac{9}{560} & \frac{17}{105} & \frac{9}{560} & \frac{9}{70} & \frac{41}{1680} & \dots & 1 & 1 \end{bmatrix}^T \begin{bmatrix} hf_n \\ hf_{n+\frac{1}{12}} \\ hf_{n+\frac{1}{6}} \\ hf_{n+\frac{1}{4}} \\ hf_{n+\frac{1}{3}} \\ hf_{n+\frac{5}{12}} \\ hf_{n+\frac{1}{2}} \\ \vdots \\ hf_{n+\frac{1}{12}} \\ hf_{n+\frac{1}{2}} \end{bmatrix}$$

4. Real Life Problems

4.1. Problem 1: (SIR MODEL)

The SIR model is an epidemiological model that computes the theoretical numbers of people infected with a contagious illness in a closed population over time. The name of this class of models derives from the fact that they involves coupled equations relating the number of susceptible people $S(t)$, number of people infected $I(t)$ and the number of people who have recovered $R(t)$. This is a good and simple model for many infectious diseases including measles, mumps and rubella [13-15]. The SIR model is described by the three coupled equations.

$$\frac{ds}{dt} = \mu(1 - S) - \beta IS \tag{11}$$

$$\frac{dI}{dt} = -\mu I - \gamma I + \beta IS \tag{12}$$

$$\frac{dS}{dt} = -\mu R + \gamma I \tag{13}$$

where μ, γ and β are positive parameters. Define y to be

$$y = S + I + R \tag{14}$$

Adding Equations (11)-(13), we obtain the following evolution equations for y

$$y' = \mu(1 - y) \tag{15}$$

Taking $\mu = 0.5$ and attaching an initial condition $y(0) = 0.5$ (for a particular closed population), we obtain,

$$y'(t) = 0.5(1 - y), y(0) = 0.5 \tag{16}$$

whose exact solution is,

$$y(t) = 1 - 0.5e^{-0.5t} \tag{17}$$

Applying our new half step numerical scheme (8) to solve SIR model simplified as (17) gives results as shown in **Table 1**.

4.2. Problem 2 (Growth Model)

Let us consider the differential equation of the form;

$$\frac{dN}{dt} = \alpha N, N(0) = 1000, t \in [0,1] \tag{18}$$

Equation (18) represents the rate of growth of bacteria in a colony. We shall assume that the model grows continuously without restriction. One may ask; how many bacteria are in the colony after some minutes if an individual produces an offspring at an average growth rate of 0.2? We also assume that $N(t)$ is the population size at time t (**Table 2**).

The theoretical solution of (18) is given by;

$$s \tag{19}$$

Note that, growth rate $\alpha = 0.2$ in (18).

Table 1. Showing results for SIR model problem.

X	Exact Result	Computed Solution	Error in half step method	Error in Sunday <i>et al.</i>
0.1	0.5243852877496430	0.5243852877496429	1.110223e-016	5.574430e-012
0.2	0.5475812909820202	0.5475812909820202	0.000000e+000	3.946177e-012
0.3	0.5696460117874711	0.5696460117874710	1.110223e-016	8.183232e-012
0.4	0.5906346234610092	0.5906346234610089	2.220446e-016	3.436118e-011
0.5	0.6105996084642976	0.6105996084642973	3.330669e-016	1.929743e-010
0.6	0.6295908896591411	0.6295908896591410	1.110223e-016	1.879040e-010
0.7	0.6476559551406433	0.6476559551406429	4.440892e-016	1.776835e-010
0.8	0.6648399769821805	0.6648399769821799	5.551115e-016	1.724676e-010
0.9	0.6811859241891134	0.6811859241891132	2.220446e-016	1.847545e-010
1.0	0.6967346701436834	0.6967346701436828	5.551115e-016	3.005770e-010

Table 2. Showing results for growth model problem.

X	Exact Result	Computed Solution	Error in half step method	Error in Sunday <i>et al.</i>
0.1	1020.2013400267558	1020.201340026755	0.000000e+000	1.830358e-011
0.2	1040.8107741923882	1040.8107741923882	0.000000e+000	1.250555e-011
0.3	1061.8365465453596	1061.8365465453596	0.000000e+000	1.227818e-011
0.4	1083.2870676749587	1083.2870676749585	2.273737e-013	3.137757e-011
0.5	1105.1709180756477	1105.1709180756475	2.273737e-013	2.216893e-010
0.6	1105.1709180756477	1127.4968515793755	2.273737e-013	2.060005e-010
0.7	1150.2737988572273	1150.2737988572271	2.273737e-013	2.171419e-010
0.8	1173.5108709918102	1173.5108709918102	0.000000e+000	2.216893e-010
0.9	1197.2173631218102	1197.2173631218102	0.000000e+000	2.744400e-010
1.0	1221.4027581601699	1221.4027581601699	0.000000e+000	4.899903e-010

Applying our new half step numerical scheme (8) to solve the Growth model (17) gives results as shown in **Table 2** [16].

4.3. Problem 3 (Decay Model)

A certain radioactive substance is known to decay at the rate proportional to the amount present. A block of this substance having a mass of 100 g originally is observed. After 40 mins, its mass reduced to 90 g. Find an expression for the mass of the substance at any time and test for the consistency of the block integrator on this problem for $t \in [0,1]$.

The problem has a differential equation of the form;

$$\frac{dN}{dt} = -\alpha N, N(0) = 100, t \in [0,1] \tag{21}$$

where N represents the mass of the substance at any time t and α is a constant which specifies the rate at which this particular substance decays. Note that,

$$f(0) = 100 \text{ g}, t = 40 \text{ mins}, f(40) = 90 \text{ g}$$

Since for any growth/decay problem,

$$f(t) = f(0)e^{\alpha t}$$

$$90 = 100e^{40\alpha}$$

$$\alpha = \frac{(\ln 9 - \ln 10)}{40} = -0.0026$$

Thus, the theoretical solution to (20) is given by,

$$f(t) = 100e^{-0.0026t} \tag{22}$$

which is also the expression for the mass of the substance at any time t .

Applying our new half step numerical scheme (8) to solve the Growth model (22) gives results as shown in **Table 3** [16,17] (**Table 3**).

5. Discussion of the Result

We have considered three real-life model problems to test the efficiency of our method. Problems 1 and 2 and 3 were solved by Sunday *et al.* [17]. They proposed an order six block integrator for the solution of first-order ordinary differential equations. Our half-step block method gave better approximation as shown in **Tables**

Table 3. Showing results for decay model problem.

X	Exact Result	Computed Solution	Error in half step method	Error in Sunday <i>et al.</i>
0.1	99.9740033797070850	99.9740033797070850	0.000000e+000	0.000000e+000
0.2	99.9480135176568470	99.9480135176568330	1.421085e-014	1.421085e-014
0.3	99.9220304120923400	99.9220304120923400	0.000000e+000	0.000000e+000
0.4	99.8960540612571460	99.8960540612571460	0.000000e+000	0.000000e+000
0.5	99.8700844633952300	99.8700844633952440	1.421085e-014	0.000000e+000
0.6	99.8441216167510670	99.8441216167510820	1.421085e-014	0.000000e+000
0.7	99.8181655195695610	99.8181655195695750	1.421085e-014	0.000000e+000
0.8	99.7922161700960970	99.7922161700960970	0.000000e+000	0.000000e+000
0.9	99.7662735665764730	99.7662735665764730	0.000000e+000	0.000000e+000
1.0	99.7403377072569700	99.7403377072569700	0.000000e+000	0.000000e+000

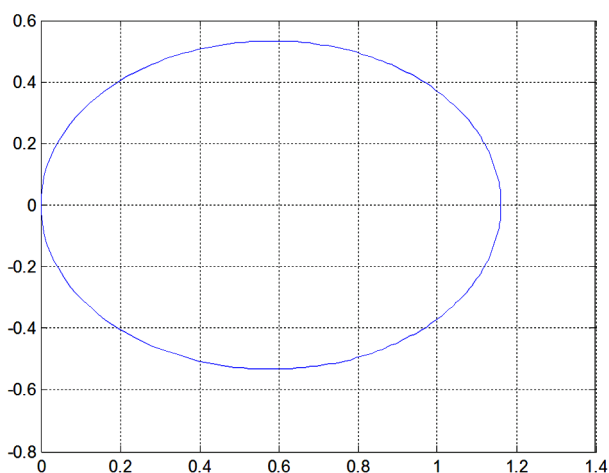


Figure 1. Showing region of absolute stability of our method.

1-3 because the iteration per step in the new method was lower than the method proposed by [17]. Our method was found to be zero stable, consistent and converges. **Figure 1** shows the region of absolute stability. From the numerical examples, we could safely conclude that our method gave better accuracy than the existing methods.

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