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Generalized Newton's Theory of Universal Gravitation and Black Holes

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Abstract

The Newton's theory of universal gravitation is generalized. Significantly strong at short distances central interaction of bodies and particles is established in comparison with Newtonian. A connection is found with Black Holes, with the horizon of events. Possibility of systematization of all Black Holes is shown. An illustration is given on the example of Black Hole $S_{gr}A^*$.

Keywords

Gravitation, Central Interaction, Escape Velocity, Black Hole, Horizon of Events

1. Introduction

The construction of the theory of movement of the solar planet system was a triumph of science in the Middle Ages. The confrontation between the supporters of Ptolemaic geocentric system and heliocentric system of Aristarchus-Copernicus, lasting for centuries, ended with victory of the latter. In the III century B.C., the representative of the Pythagorean School of Greece, Aristarchus of Samos (310-230 B.C.) has advanced the heliocentric system of planetary motion. According to this model, in the center of the Universe is situated the Sun, which is motionless and around it revolves the Earth and other planets. This model was rejected by ancient astronomers as baseless, in their opinion, since it required the moving of Earth around the Sun. In future, the most famous was the geocentric system of Ptolemy (*Claudius Ptolemy*, 90-168 A.D.), who lived in Alexandria in the II century A.D. In Ptolemy system, everything is explained with the help of circumferences and circular motions.

It is believed that the planet moves along an epicycle (circle), center of which, in its turn, moves along deferent (another circle), center of which coincides with the Earth. Ptolemy's "Almagest" book (in Arabic means the greatest) contained everything to calculate the position of the Sun, the Moon and planets for every night. From the observations of the starry sky, ancients concluded that it goes around our Earth, which is motionless and located at the center of the Universe. Over time, the accuracy and reliability of astronomical observations increased, especially after the invention of the telescope by Galilei (*Galileo Galilei*, 1564-1641). His observations showed that around other planets also moves along celestial bodies. This contradicted Ptolemy's theory. The improvement of this theory was reduced to the assumption that additional epicycles move along epicycles, which greatly complicated the system and it was in danger of falling apart under the weight of its own complexity.

Eighteen centuries after Aristarchus, Copernicus (*Nicolaus Copernicus*, 1473-1543) revived the heliocentric model and unlike Aristarchus, who only expressed the general idea, developed the details of the heliocentric model and basics for calculating positions of planets.

Yet, Copernicus went on relying on Ptolemy's method of the circular orbits. According to his theory, all the planets move around the Sun in circular orbits and uniformly.

The radical change in the victory of the heliocentric system introduced Johann Kepler (1571-1630) at the beginning of the XVII century. Based on wonderful catalog of exclusively exact observations, connected with planetary motion, composed by Tycho Brahe (1546-1601), in particular data of Mars, Kepler formulated his three famous laws.

According to established by Kepler first law, any planet of the solar system moves around the Sun in an elliptical orbit with the Sun in one of focuses of the ellipse. With this, the Ptolemean geocentrical model of motion was disproved.

According to the second law, any planet moves in the orbit with a constant sectorial velocity, *i.e.* the straight line, connecting the planet with the Sun, outlines equal areas during equal time interval.

Kepler's third law establishes the connection between the big semi-axis (a) of the ellipse and the period (T) during which the planet completes a full turn:

$$\frac{T^2}{a^3} = \frac{4\pi^2}{G(M+m)},$$

G-gravitational constant.

Since Kepler's laws are established on base of subtle analysis of excellent measurement data, they are taken as empirical.

Several decades later Newton (*Isaac Newton*, 1642-1717) mathematically derived Kepler's laws and formulated the famous gravity law. According to this law the force of the gravity is central and each mass m is gravitated by another mass M in the Universe with force unversely proportional to the square of the distance between the masses and is directed along the line, connecting the centers of the masses. Newton's another important achievement was that he proved that the orbit of the bodies moving around the Sun, may be any of the curves of the conical sections family: circle, ellipse, parabola, hyperbola.

In the next decades and centuries Newton's gravity law has received a lot of convincing and vivid confirmations [1] [2]. Edmond Halley (1656-1742) based on Newton's gravity law predicted the next appearance in the sky of Earth (December, 1758) observed since ancient times (240 B.C.) the comet Halley (called by his name in 1759; it appears in the Sky of the Earth with the period of 75 - 76 years). Last time Halley's Comet passed Earth on February 9, 1986, the coming next appearance is expected on July 28, 2061. It is interesting, that Halley did not live up to the date of his prediction. Yet his prediction became the first successful confirmation of Newton's Celestial Mechanics and clear demonstration of its predictive power. Leaning on Newton's law William Herschel (1738-1822) opened the planet Uranus (1781). English mathematician and astronomer Adams (*John Adams*, 1819-1892) and French astronomer and mathematician Le Verrier (*Jean Le Verrier*, 1811-1877) independently from each other calculated it position and in 1846 opened the planet Neptune. German astronomer Galle (*Johann Galle*, 1812-1910) discovered the planet in the sky in the place, indicated by Le Verrier. American young astronomer from the Lowell Observatory, Clyde Tombo (*Tombaugh*, *Clyde William*, 1906-1997) on February 18, 1930 discovered the planet Pluto. American scientists James, L. Elliot, Edward, W. Dunham, Douglas, J. Mink on March 10, 1977 discovered the rings (it is 13 of them) of Uranus.

Along with many successes, over time, were revealed cosmic phenomena that could not be explained by Newton's theory of gravity. In 1859 Le Verrier detected some discrepancy in the orbit of the planet, closest to the Sun, Mercury at perigee with the observations results [2] [3]. Not finding any convincing explanations of this fact, in 1895, well-known American astronomer Simon Newcomb (1835-1909), expressed an opinion, that possibly Newton's law of inverse squares was not performed precisely at small distances. An attempt was made to explain the arose problem with Mercury on the base of the General theory of relativity (GTR), but it did not give the desired results [2].

Another very important problem arose, satisfying solution to which has not been found yet. This problem is about "Black Holes".

English astronomer-amateur, one of the founders of Seismology John Mitchell (1724-1793) and well-known French Mathematician and Mechanics Laplass (*Pierre Laplass*, 1749-1827), independently of each other on the base of Newton's theory of gravity expressed an opinion that in nature should exists bodies for which the necessary velocity for overcoming their gravitation exceeds the speed of light (c). Therefore such body, which latter called "Black Hole" should be "dark" *i.e.* invisible for not even light can leave it. It can be discovered indirectly by gravitational influence on other bodies.

Mitchell and Laplass, using the idea of the second cosmic velocity (escape velocity) derived the radius of "Dark Body" (gravitational radius):

$$r_g = \frac{2GM}{c^2} \tag{1}$$

where M-mass of the "Dark body".

According to the General Theory of Relativity (GTR), by passage to the limit the same value of the gravitational radius r_g was obtained. Supporters of the GTR were criticized Mitchell and Laplace in the sense that at the speeds close to the speed of light velocity formulas of classical Mechanics are not applicable. Opponents of the GTR claim that this theory it is not applicable for discribing macroscopic phenomena in Universe, because the solution of equations of this theory contains singularity unacceptable for discribing natural phenomena. Moreover, planets and stars, as a rule, do not have significant velocities comparable to the speed of light, and relativistic effects for them are unessential [4].

To bring justified clarity, the presence of more General theory of universal gravitation becomes necessity. The New theory of gravitation should not contradict Newton's theory, because it has received multiple confirmations long ago, and should provide more opportunities for explaining at least some of the phenomena which are difficult to explain by Newton's theory.

2. Generalization of Newton's Theory of Universal Gravitation

In author works [5] [6] [7] it was proved the possibility of generalization of Newtonian theory of universal gravitation. The new central interaction of bodies and particles is more powerful at short distances than Newtonian. The trajectory of motion can again be a conic section. To this interaction, in particular, is subjected "Black Holes".

Let us have bodies with masses m, M. Put the beginning of polar coordinates (r,θ) at the center of the body with mass M. The central force of the interaction we will be given in the form

$$\boldsymbol{F} = -GmM\frac{e^{k/r}}{r^2}\frac{\boldsymbol{r}}{|\boldsymbol{r}|} \tag{2}$$

or

$$F = -GmM\frac{e^{k/r}}{r^2} \tag{3}$$

where G, gravitational constant in Newton's law of universal gravitation ($G = 6.67\times10^{-11}\,\mathrm{m}^3/(\mathrm{kg}\cdot\mathrm{s}^2)$). Index "$k$" will characterize the power (intensity) of the center of attraction. At $k=0$ interaction (2), (3) coincides with the Newtonian one ($F=-GmM/r^2$). Taking this into account, in (2) was introduced the coefficient GmM. And if $k>0$, the interaction will be more powerful than the Newtonian. It is obvious that "k" has the dimensionality of the length, we will speak about its possible values a bit later.

The field made by force $\boldsymbol{F}$ given in Formula (2) is potential, with potential

$$U = -\int F\mathrm{d}r = -\frac{GmM}{k}e^{\frac{k}{r}} + const, \tag{4}$$

which is essentially stronger than the potential of Newton field ($U=-\frac{GmM}{r}$), especially at short distances from the center of attraction.

As force $\boldsymbol{F}$ is central, the trajectory of the point is plane curve and the law of constancy of sectorial velocity takes place (the law of the squares):

$$r^2 \frac{\mathrm{d}\theta}{\mathrm{d}t} = C \tag{5}$$

where C, is moment of initial velocity relatively to the center of gravity.

Taking into account (5), the velocity of point in system of polar coordinates will have the form

$$v^2 = C^2 \left[\left(\frac{\mathrm{d}\frac{1}{r}}{\mathrm{d}\theta} \right)^2 + \left(\frac{1}{r} \right)^2 \right] \tag{6}$$

On the other hand, according to the theorem of kinetic energy: $\mathrm{d}\frac{mv^2}{2} = \boldsymbol{F}\mathrm{d}\boldsymbol{r}$, taking into account (2), (3) we will have

$$v^2 = \frac{2GM}{k} \mathrm{e}^{k/r} + h \tag{7}$$

where the integration constant h is determined from the initial conditions:

$$\text{at} \quad r = r_0, \quad v = v_0 \tag{8}$$

Using (6), (7) we determine the trajectory of motion [6]

$$r = \frac{1/k_1}{1 + (k_2/k_1)\cos\sqrt{\delta_1}(\theta - \theta_0)} \tag{9}$$

where

$$\begin{gathered} \delta_1 = 1 - \frac{GMk}{C^2}, \quad k_1 = \frac{GM}{C^2 \delta_1}, \quad \delta_1 > 0 \\ k_2^2 = k_1^2 + (2GM + kh) \frac{1}{kC^2 \delta_1} = \frac{1}{kC^2 \delta_1^2} \left[GM + (GM + kh) \delta_1 \right] \end{gathered} \tag{10}$$

According to (9), the trajectory is conic section, with parameters

$$\begin{gathered} p = \frac{1}{k_1} = \frac{C^2}{GM} - k, \\ \varepsilon = \frac{k_2}{k_1} = \sqrt{1 + \left(\frac{C^2}{MGk} - 1 \right) \left(2 + \frac{hk}{MG} \right)} \end{gathered} \tag{11}$$

The trajectory is an ellipse at

$$-\frac{MG}{k} \left(1 + \frac{C^2}{C^2 - MGk} \right) < h < -\frac{2GM}{k} \tag{12}$$

parabola, at $h = -(2GM)/k$, hyperbola, at $h > -(2GM)/k$.

In the case of an ellipse, the semi-axes are determined by the formulas

$$a = \frac{p}{1 - \varepsilon^2}, \quad b = \frac{p}{\sqrt{1 - \varepsilon^2}} \tag{13}$$

Using conditions (8) from (7), we determine the value of h:

$$h = v_0^2 - \frac{2GM}{k} \mathrm{e}^{k/r_0} \tag{14}$$

According to (14), the last inequality (12) can be written in the form:

$$v_0^2 < v_*^2, \quad v_*^2 = \frac{2GM}{r_0} \frac{\mathrm{e}^{k/r_0} - 1}{k/r_0} \tag{15}$$

From Formula (15) it follows, that at the initial speed $v_0 < v_*$, the trajectory is an ellipse, at $v_0 = v_*$, parabola, at $v_0 > v_*$, hyperbola.

$\lim\limits_{k \to 0} v_*^2 = \frac{2GM}{r_0} = v_N^2$, $v_N = \sqrt{\frac{2GM}{r_0}}$, well-known escape velocity (the second cosmic velocity) by Newton's theory. At $k > 0$ $v_* > v_N$, *i.e.* the escape velocity under the interaction (2) is greater than the escape velocity according to Newton's theory, and this difference can be arbitrarily large. This, in its turn, means that in order to overcome the attraction of the field, corresponding to interaction (2), super high velocities will be required.

3. Black Hole and Its Gravitational Radius

The body with mass M will be Black Hole ("dark", invisible), if anybody with mass $m < M$ and initial velocity, even equal to the speed of light "c", cannot overcome the field of gravitation of mass M. Let us find out what will be the gravitational radius R_g during the interaction (2).

The initial conditions of the problem will be:

$$\text{at } r_0 = R_g, \quad v_0 = c \tag{16}$$

According to (15), (16) we will have

$$\frac{2GM}{R_g} \frac{\mathrm{e}^{\frac{k}{R_g}} - 1}{\frac{k}{R_g}} = c^2 \tag{17}$$

Noting $\lim\limits_{k \to 0} R_g = r_g$, we will calculate in (17) the limit at $k \to 0$. As a result we will have

$$\frac{2GM}{r_g} = c^2 \quad \text{or} \quad r_g = \frac{2GM}{c^2} \tag{18}$$

i.e. r_g is well-known gravitational radius (1) for Newton's central interaction.

Noting $\gamma = k/R_g$. Then Formula (17), taking into account (18), can be written in the form

$$R_g = r_g \frac{\mathrm{e}^\gamma - 1}{\gamma} \quad \text{or} \quad \frac{R_g}{r_g} = \frac{\mathrm{e}^\gamma - 1}{\gamma} \tag{19}$$

The Formula (19) reflects the relationship between the gravitational radii of a body (formation) of mass M according to the generalized Newton's theory and Newton's theory of gravitation. From this formula it follows that R_g can be arbitrarily more than r_g (see graph of function R_g/r_g, **Figure 1**). Having the

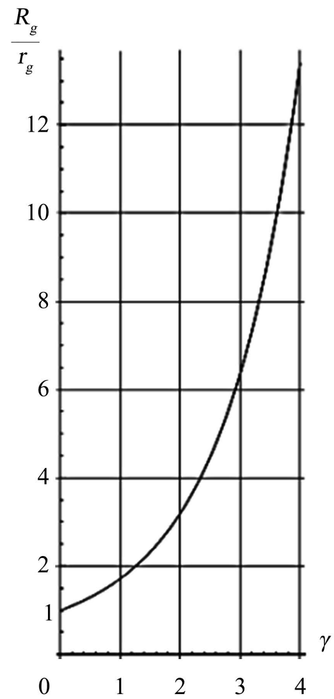

Figure 1. The dependence between R_g/r_g and the intensity index γ of gravitational field of Black Hole.

value ratio R_g/r_g, from Equation (19), will be determined the intensity index γ of the center of attraction, as well as intensity $k=\gamma R_g$. Conversely, by setting the value of γ from the same formula, it will be determined R_g/r_g. From the graph of function R_g/r_g and Equation (19) it follows, that a lot of Black Holes may exist. This fact has long ago been confirmed by astronomers and astrophysicists. They suppose that every Galaxy has at least one Black Hole, which approximately located in the center of the Galaxy.

There is enough extensive catalog of Black Holes. Let us point some of the supposed Black Holes: NGC 6166 [8]; S 50014 + 81 [9]; Sgr A* [10] [11] [12]; NGC 1277 [13]; Swan A [14]; OJ 287 second [15]; Margaryan 771 [16]; Messie 84 [17]; NGC 1281 [18]; NGC 1399 [19]); NGC 3115 [20]; TON 618 (Canes Venatici) [21]; OJ 287 principal [15]; Margaryan 79 [16].

According to Newton's theory of gravitation, by Formula (18) determines the gravitational radius of an arbitrary body of mass *M*. For the planet Earth $M=5.9736\times10^{24}\ \text{kg}$, $r_g=8.87\ \text{mm}\approx 9\ \text{mm}$, for the Sun, mass usually denoted by $\odot$ and $\odot=2\times10^{30}\ \text{kg}$, $r_g=2.95\ \text{км}\approx 3\ \text{km}$. *I.e.* the Earth can become Black Hole if in some way its whole mass is succeeded to be put into a ball with radius of 9 mm, for the Sun, into a ball with radius of 3 km, which is not real to imagine.

The mass of any Black Hole is in multiple times greater than the mass of the Sun.

In **Table 1** were given the values of masses and Newton's of gravitational radii of the above mentioned Black Holes.

Table 1. Newton's gravitational radius of some supposed Black Holes.

Black Holes	*Mass*	*Gravitational radius* r_g (*km*)
NGC 6166	$3\times10^{10}\odot$	9×10^{10}
S 50014 + 81	$4\times10^{10}\odot$	12×10^{10}
Sagittarius A^* ($S_{gr}A^*$)	$4.3\times10^{6}\odot$	12.9×10^{6}
NGC 1277	$1.2\times10^{9}\odot$	3.6×10^{9}
Swan A	$1\times10^{9}\odot$	3×10^{9}
OJ 287 second	$1\times10^{8}\odot$	3×10^{8}
Margaryan 771	$7.586\times10^{7}\odot$	22.758×10^{7}
Messie 84	$1.5\times10^{9}\odot$	4.5×10^{9}
NGC 1281	$1\times10^{10}\odot$	3×10^{10}
NGC 1399	$5\times10^{8}\odot$	15×10^{8}
NGC 3115	$2\times10^{9}\odot$	6×10^{9}
TON 618 The constellation of Hound Dogs (*Canes Venatici*)	$6.6\times10^{10}\odot$	19.8×10^{10}
OJ 287 principal	$1.8\times10^{10}\odot$	5.4×10^{10}
Margaryan 79	$5.25\times10^{7}\odot$	15.75×10^{7}

4. Horizon of Events and Systematization of Black Holes

The surface around the Black Hole, which other bodies (particles) cannot overcome yet, having speed even equal to the speed of light is called the "horizon of events". This term was introduced into physics and astrophysics by the Austrian-American physicist Wolfgang Rindler (*Wolfgang Rindler*, 1924-2019) in 1956. By the famous astrophysicist of our times Stephen Hawking (*Stephen Hawking*, 1942-2018) the horizon of events is made of light, which is not able to leave the Black Hole and that is why "soar" on this horizon [22].

The surface, that serves as the horizon of events is usually considered to be spherical, the radius of this surface is the gravitational radius R_g of Black Hole or radius of Schwarzschild (*Karl Schwarzschild*, 1873-1916) by name of the famous German astronomer. It is geometrically obvious that the Schwarzschild's gravitational radius R_g is greater than the Newton's gravitational radius r_g.

Let that in some way was determined the radius of the horizon of events (Schwarzschild's radius) R_g of a known (assumed) Black Hole. Then, from the known mass of that Black Hole, at first we calculate Newton's gravitational radius r_g ($2GM/c^2$). For some Black Holes in **Table 1** are given the corresponding values of r_g. As a result, the value R_g/r_g becomes also known. Then from the graph of function R_g/r_g (**Figure 1**) or from the Equation (19) the value of intensity index γ of the force field of Black Hole is immediately unambiguously determined, as well as the intensity $k=\gamma R_g$. Because of this, becomes very relevant issue of determining not only masses of Black Holes, but also the determination of value of radius R_g of horizon of events.

Let us discuss this issue in more detail for Black Hole Sagittarius A^* ($S_{gr}A^*$). It located in the center of our Galaxy Milky Way, [10] [12] [23] [24]. The object was first discovered by Robert Brown (1946-2002) and Bruce Balick in National Radio-Astronomy Observatory (USA) in 1974 on 13 and 15 of February. Subsequently, research were carried out on the basis of observations of the movement of star *S*2 around supposed Black Hole $S_{gr}A^*$. During the period of observations (1992-2021), star *S*2 made almost two complete revolution around Black Hole, which made it possible to determine with great accuracy parameters of its orbit.

The period of revolution of *S*2 is (15.8 ± 0.11) years, the big semi-axis of the orbit ≈1000 a.u., the eccentricity – 0.88441 ± 0.00006, the maximum approach to $S_{gr}A^*$ -119.54 a.u. [25].

$S_{gr}A^*$ is surrounded by gas radio-emitting hot cloud with diameter about 1.8 parsec. The diameter of $S_{gr}A^*$ is 44 million km, for comparison, the distance between the Sun and Mercury is 46 million km [24].

In 2004 was discovered the cluster of seven stars, which revolve around $S_{gr}A^*$ by orbit with distance of three light years. All this facts is the best evidence of existence of supermassive Black Hole $S_{gr}A^*$.

Roger Penrose, UK; Reinhard Genzel, Germany; Andrea Ghez, USA were awarded with Nobel Prize in the field of Physics in 2020 with the following formulation: "For the discovery supermassive compact object in the center of our Galaxy".

In the most of publications, it is accepted that the mass of $S_{gr}A^*$ is $(4.297 \pm 0.042)\times 10^6 \odot$. Radius of horizon of evens (Schwarzschild's radius) for $S_{gr}A^*$ is 45 a.u. (6732 million km). Naturally, it is larger than the linear characteristic size of (radius *R*) $S_{gr}A^*$, which is 22 million km (half of the diameter of $S_{gr}A^*$). Thus, we have,

$$R_g = 45\ \text{a.u.} = 45\times 149597871\ \text{km} \approx 6732\times 10^6\ \text{km}, \quad r_g = 12.9\times 10^6\ \text{km}.$$

According to **Table 1** for $S_{gr}A^*$ Newton's gravitational radius $r_g = 12.9\times 10^6\ \text{km}$. Consequently

$$\frac{R_g}{r_g} = \frac{6732}{12.9} = 521.860465 \approx 522 \tag{20}$$

Using (20) from the Equation (19) we uniquely determine the value of intensity index γ of force field of $S_{gr}A^*$. We will have $\gamma = 8.38395$. Consequently

$$k = \gamma R_g = 8.38395\times 6732\times 10^6\ \text{km} = 56440.7514\times 10^6\ \text{km}. \tag{21}$$

The force field created by Black Hole $S_{gr}A^*$, according to (2) and (21) will have the form

$$\boldsymbol{F} = -GmM\frac{\mathrm{e}^{8.38395\frac{R_g}{r}}}{r^2}\frac{\boldsymbol{r}}{|\boldsymbol{r}|} \tag{22}$$

Taking into account, that R_g for $S_{gr}A^*$ is an immense number ($R_g = 45\ \text{a.u.}$), from (22) it follows, that the gravitational force field at short distances is super strong, forcing to revolve around $S_{gr}A^*$ as well stars.

The above stated procedure remains true for all Black Holes. Each Black Hole creates near itself the gravitational field which is exponentially stronger than by Newton's gravitational theory:

$$\boldsymbol{F} = -GmM\frac{e^{\gamma\frac{R_g}{r}}}{r^2}\frac{\boldsymbol{r}}{|\boldsymbol{r}|} \tag{23}$$

where R_g -is radius of horizon of events, γ, the intensity index of the gravitational field of Black Hole, which is determined from the Equation (19).

All Black Holes can be systematized according to their mass, radius of horizon of events R_g (Schwarzchild's radius) and gravitational intensity index γ of field, created by Black Hole. In both cases, the radius R_g of horizon of events must be known.

5. Discussion and Conclusions

The central interaction of bodies of universal gravitation is established, which at short distances is exponentially stronger than Newton's interaction. Conditions are derived when the trajectory of a body in such interaction is conical section. The escape velocity (second cosmic velocity) is determined, which is much greater than Newton's escape velocity.

It is shown that Black Hole is subordinated to new central interaction

$$\boldsymbol{F} = -GmM\frac{e^{\gamma\frac{R_g}{r}}}{r^2}\frac{\boldsymbol{r}}{|\boldsymbol{r}|}$$

where R_g, radius of horizon of events (Schwarzchild's radius), γ, the intensity index of the gravitational field of Black Hole, which is satisfying equation:

$$\frac{R_g}{r_g} = \frac{e^{\gamma} - 1}{\gamma}$$

where r_g, gravitational radius by Newton's theory; is shown the possibility of systematization of all Black Holes according to their mass, radius of horizon of events R_g and gravitational intensity index γ of field of gravitation. In particular, for Black Hole $S_{gr}A^*$ we have:

$$M = 4.3\times10^6\odot,\quad r_g = 12.9\times10^6\ \text{km},\quad \odot = 2\times10^{30}\ \text{kg}$$

$$R_g = 45\ \text{a.u.} \approx 6732\times10^6\ \text{km},\quad \gamma = 8.38395$$

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Kittel, C., Knight, W.D., *et al.* (1965) Mechanics (Berkley Physics Course, Vol.1).

McGraw-Hill, New York.

[2] Layzer, D. (1984) Constructing the Universe. Scientific American Book. Inc., New York.

[3] Roseveare, N.T. (1982) Mercury's Perihelion from Le Verrier to Einstein. Clarendon Press, Oxford.

[4] Bergman, P.G. (1968) The Riddle of Gravitation. Charles Scribner's Sons, New York.

[5] Aghalovyan, L.A. (2006) On Universal Central Strong (Weak) Interaction of Bodies and Particles. *Reports of National Academy of Sciences of Armenia*, **106**, 238-244.

[6] Aghalovyan, L.A. (2018) The Consequences of a Strengthened Newtonian Gravity at Short Distances. *International Journal of Astronomy and Astrophysics*, **8**, 191-199. https://doi.org/10.4236/ijaa.2018.82014

[7] Aghalovyan, L.A. (2020) Consequences of a Generalized Newtonian Gravity with an Exsponential Factor. *International Journal of Astronomy and Astrophysics*, **10**, 224-234. https://doi.org/10.4236/ijaa.2020.103012

[8] Magorrian, J., Tremaine, S., *et al.* (1998) The Demography of Massive Dark Objects in Galaxy Centeres. *The Astronomical Journal*, **115**, 2285-2305. https://doi.org/10.1086/300353

[9] Ghiselini, G., Ceca, R., *et al.* (2010) Chasing the Heaviest Black Holes in Active Galactic Nuclei, the Largest Black Hole. *Monthly Notices of the Royal Astronomical Society*, **405**, 387-400. https://doi.org/10.1111/j.1365-2966.2010.16449.x

[10] Eckart, A., Schodel, R., *et al.* (2005) The Black Hole at the Center of Milky Way. Imperial College Press, London. https://doi.org/10.1142/p394

[11] Ghez, A., Salim, S., *et al.* (2005) Stellar Orbits around the Galactic Center Black Hole. *The Astrophysical Journal*, **620**, 744-757. https://doi.org/10.1086/427175

[12] Ghez, A., Salim, S., *et al.* (2008) Measuring Distance and Properties of the Milky Way's Central Supermassive Black Hole with Stellar Orbits. *The Astrophysical Journal*: *Journal*, **689**, 1044-1062. https://doi.org/10.1086/592738

[13] Graham, A.W., Durre, M., *et al.* (2016) A Normal Supermassive Black Hole in NGC 1277. *The Astrophysical Journal*, **819**, Article ID: 43. https://doi.org/10.3847/0004-637X/819/1/43

[14] Black Holes (2015) Gravity's Relentless Pull Interactive Encyclopedia-Hubble Site. https://www.stsci.edu/~marel/black_holes/encyclopedia.html

[15] Valtonen, M., Ciprini, S., *et al.* (2012) On the Masses of OJ 287 Black Holes. *Monthly Notices of the Royal Astronomical Society*, **427**, 77-83. https://doi.org/10.1111/j.1365-2966.2012.21861.x

[16] Nelson, C.H. (2000) Black Hole Mass, Velocity Dispersion, and the Radio Source in Active Galactic Nuclei. *The Astrophysical Journal*, **544**, L91-L94. https://doi.org/10.1086/317314

[17] Bower, G., Green, R., *et al.* (1998) Kinematics of the Nuclear lonized Gas in the Radio Galaxy M 84 (NGC 4374). *The Astrophysical Journal*, **492**, L111-L114. https://doi.org/10.1086/311109

[18] Vildirim, A., Bosch, V., *et al.* (2016) The Massive Dark Hole of the Compact Early-Type Galaxy NGC 1281. *Monthly Notices of the Royal Astronomical Society*, **456**, 538-553. https://doi.org/10.1093/mnras/stv2665

[19] Gebhardt, K., Zauer, T.R., *et al.* (2007) The Black Hole Mass and Extreme Orbital Structure in NGC 1399. *The Astrophysical Journal*, **671**, 1321-1328.

https://doi.org/10.1086/522938

[20] Kormendy, J. and Richstone, D. (1992) Evidence for a Supermassive Black Hole in NGC 3115. *The Astrophysical Journal*, **393**, 559-578. https://doi.org/10.1086/171528

[21] Shemmer, O., Netzer, H., *et al*. (2004) Near-Infrared Spectroscopy of High-Red Shift Active Galactic Nuclei. I. A Metallicity-Accretion Rate Relationship. *The Astrophysical Journal*, **614**, 547-557. https://doi.org/10.1086/423607

[22] Hawking, S. (1996) Black Holes and Baby Universes and Other Essays. Bantam Book, Toronto.

[23] Melia, F. (2003) The Black Hole in the Center of Our Galaxy. Princeton University Press, Princeton. https://doi.org/10.1515/9780691186368

[24] Abuter, R., *et al*. (2021) Mass Distribution in the Galuctic Center Based on Inter Forometric Astrometry of Multiple Stellar Orbits. *Astronomy & Astrophysics*, **657**, Article ID: L12.

[25] Gillessen, S., Eisenhauer, F., *et al*. (2009) Monitoring Stellar Orbits around the Massive Black Hole in the Galactic Centre. *The Astrophysical Journal*, **692**, 1075-1109. https://doi.org/10.1088/0004-637X/692/2/1075