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On $\tau_1\tau_2$-$\bar{g}$-Open Sets in Bitopological Spaces

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Authors' contributions

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Abstract

In this paper, we introduced and studied a new kind of generalized open set called $\tau_1\tau_2$-$\bar{g}$-open set in a bitopological space (X, τ_1, τ_2). The properties of this $\tau_1\tau_2$-$\bar{g}$-open set are studied and compared with some of the corresponding generalized open sets in general topological spaces and bitopological spaces. We also defined the $\tau_1\tau_2$-$\bar{g}$-continuous function and studied some its properties.

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1 Introduction

A space X equipped with two arbitrary topologies τ_1 and τ_2 is defined by J. C. Kelley [1] as the bitopological space in 1963 and denoted it by a triple (X, τ_1, τ_2) to generalize a topological space (X, τ). Every bitopological space (X, τ_1, τ_2) can be regarded as a topological space (X, τ) if $\tau_1 = \tau_2 = \tau$. A topological space occurs for every metric spaces but the bitopological spaces occurs for quasi-metric spaces. A subset A of a bitopological space (X, τ_1, τ_2) is called open if A is both τ_1-open and τ_2-open. In mathematics, and more specifically in topology, an open set is an abstract concept generalizing the idea of an open interval in the real line. The open sets play some role in properties of topological spaces such as once a choice of open sets is made, the properties of continuity, connectedness, and compactness, which use notions of nearness, can be defined using these open sets. The different forms of open sets were studied in past few years. Levine [2] defined that the complement of g-closed set is a g-open set in 1970. A. Csaszar extended a significant contribution to the theory of generalized open sets recently. There were many different kind of generalized open sets on topological spaces and on bitopological spaces introduced by different authors. As an example, Bhattacharyya and Lahiri [3], Maki, Devi and Balachandran [4] and Keskin and Noiri [5] have introduced and studied sg-open sets, $g\alpha$-open sets and bg-open sets. In this paper, we introduce another kind of generalized open set in the bitopological space and compare this with some of the corresponding generalized open sets and then analyzed its properties.

2 Preliminaries

Throughout this paper, we represent X and Y as the bitopological spaces (X, τ_1, τ_2) and (Y, σ_1, σ_2) on which no separation axioms are assumed unless otherwise stated. For a subset of A of X, τ_i-cl(A) denotes the closure of A and τ_i-int(A) denotes the interior of A, respectively with respective to the topology τ_i.

In the topological space (X, τ), we recall the following closed sets.

Definition 2.1. A subset A of a topological space (X, τ) is called a

1. regular closed [6] if $A \subseteq \text{cl}(\text{int}(A))$.
2. ω-closed [7] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is semi open in τ.
3. semi closed [8] if $\text{int}(\text{cl}(A)) \subseteq A$.
4. α-closed [9] if $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$.

The complements of the above mentioned closed sets are their respective open sets.

The semi interior (respectively, α-interior, semi pre interior, δ-interior and $b^\sharp$-interior) of a subset A of a space (X, τ) is the union of all semi open(respectively, α-open, semi pre open, δ-open and $b^\sharp$-open) sets contained in A and is denoted by sint(A) (respectively, αint(A), spint(A), int_δ(A) and $b^\sharp$int(A)).

We also recall some generalized closed sets defined in a topological space (X, τ).

Definition 2.2. A subset A of a topological space (X, τ) is called a

1. g-closed [2] (generalized closed) if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in τ.
2. $\tau_1\tau_2$-$\bar{g}$-closed [10] if τ_i-cl$(A) \subseteq U_i$ whenever $A \subseteq U_i$ and U_i is τ_i-open for each $i = 1, 2$.
3. gs-closed [11] (generalized semi closed) if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in τ.

4. r-g-closed [12] (regular generalized closed) if $\mathrm{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is a regular open in τ.
5. αg-closed [13] (generalized semi pre closed) if $\alpha\mathrm{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in τ.
6. gp-closed [14] (generalized semi closed) if $\mathrm{pcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in τ.
7. gsp-closed [15] (generalized semi pre closed) if $\mathrm{spcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in τ.
8. $rgb^\sharp$-closed [16] (regular generalized $b^\sharp$ closed) if $b^\sharp\mathrm{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is a regular open in τ.
9. $\hat{g}$-closed [17] if $\mathrm{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is a semi open in τ.
10. $\delta\hat{g}$-closed [18] if $\mathrm{cl}_\delta(A) \subseteq U$ whenever $A \subseteq U$ and U is a $\hat{g}$-open in τ.
11. $(gsp)^*$-closed [19] if $\mathrm{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is a (gsp)-open in τ.

The complements of the above mentioned generalized closed sets are their respective generalized open sets.

Now we recall some generalized closed sets in a bitopological space (X, τ_1, τ_2).

Definition 2.3. A subset A of a bitopological space (X, τ_1, τ_2) is called a

1. $\tau_1\tau_2$-g-closed [20] ($\tau_1\tau_2$-generalized closed) if τ_2-$\mathrm{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1-open.
2. $\tau_1\tau_2$-sg-closed [21] ($\tau_1\tau_2$-semi generalized closed) if τ_2-$\mathrm{scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1-semi open.
3. $\tau_1\tau_2$-gs-closed [22] ($\tau_1\tau_2$-generalized semi closed) if τ_2-$\mathrm{scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1- open.
4. $\tau_1\tau_2$-αg-closed [23] ($\tau_1\tau_2$-α-generalized closed) if τ_2-$\alpha\mathrm{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1- open.
5. $\tau_1\tau_2$-gα-closed [23] ($\tau_1\tau_2$-generalized α-closed) if τ_2-$\alpha\mathrm{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1-α-open.
6. $\tau_1\tau_2$-$\hat{g}$-closed [22] if τ_2-$\mathrm{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1- semi open.

The complements of the above mentioned generalized closed sets in bitopological spaces are their respective generalized open sets in the corresponding bitopological spaces.

Definition 2.4. Let τ_1 and τ_2 be two topologies on a set X such that τ_1 is contained in τ_2. Then, the topology τ_1 is said to be a coarser (weaker or smaller) topology than τ_2.

3 Generalized $\tau_1\tau_2$-$\bar{g}$-Open Sets

Definition 3.1. A subset A of a bitopological space (X, τ_1, τ_2) is called a $\tau_1\tau_2$-$\bar{g}$-open if $F_i \subseteq \tau_i$-$\mathrm{int}(A)$ whenever $F_i \subseteq A$ and F_i is τ_i-closed for each $i = 1, 2$.

Example 3.2. Let $X = \{a, b, c\}, \tau_1 = \{\phi, \{a, b\}, X\}$, and $\tau_2 = \{\phi, \{b\}, \{a, c\}, X\}$. Then, $\phi, \{a\}, \{b\}, \{a, b\}$ and X are the $\tau_1\tau_2$-$\bar{g}$-open sets in (X, τ_1, τ_2).

Theorem 3.3. The intersection of two $\tau_1\tau_2$-$\bar{g}$-open sets is a $\tau_1\tau_2$-$\bar{g}$-open set.

Proof. Let A and B be two $\tau_1\tau_2$-$\bar{g}$-open sets. Then, $F_i \subseteq \tau_i$ int(A) whenever $F_i \subseteq A$ and F_i is τ_i-closed for each $i = 1, 2$ and $F_i \subseteq \tau_i$-int(B) whenever $F_i \subseteq A$ and F_i is τ_i-closed for each $i = 1, 2$. Then, we have $F_i \subseteq \tau_i$-int$(A \cap B)$ whenever $F_i \subseteq (A \cap B)$ and F_i is τ_i-closed for each $i = 1, 2$. Therefore, $A \cap B$ is a $\tau_1\tau_2$-$\bar{g}$-open set. □

The union of two $\tau_1\tau_2$-$\bar{g}$-open sets need not be a $\tau_1\tau_2$-$\bar{g}$-open set. This can be seen from the following example.

Example 3.4. Let $X = \{a, b, c\}, \tau_1 = \{\phi, \{a\}, X\}$, and $\tau_2 = \{\phi, \{b\}, X\}$. If $A = \{a\}$ and $B = \{c\}$, then the sets A and B are $\tau_1\tau_2$-$\bar{g}$-open; but, $A \cup B = \{a, c\}$ is not a $\tau_1\tau_2$-$\bar{g}$-open set.

Theorem 3.5. Every $\tau_1\tau_2$-$\bar{g}$-open set in (X, τ_1, τ_2) is a gs-open set in both τ_1 and τ_2.

Proof. Let A be any $\tau_1\tau_2$-$\bar{g}$-open set in (X, τ_1, τ_2) and F_i be any closed set in (X, τ_i) contained in A for $i = 1, 2$ respectively. Then, $F_i \subseteq \tau_i$-int(A) whenever $F_i \subseteq A$ and F_i is closed set in (X, τ_i) for $i = 1, 2$. Since τ_i-int $(A) \subseteq \tau_i$-sint (A), $F_i \subseteq \tau_i$-sint(A) whenever $F_i \subseteq A$ and F_i is closed set in (X, τ_i) for $i = 1, 2$. Therefore, A is a gs-open set in both τ_1 and τ_2. □

The converse of the above theorem need not be true as seen from the following example.

Example 3.6. Let $X = \{a, b, c\}, \tau_1 = \{\phi, \{a, b\}, X\}$ and $\tau_2 = \{\phi, \{b\}, \{a, c\}, X\}$. Then, the set $\{c\}$ is a gs-open set in both τ_1 and τ_2. But, it is not a $\tau_1\tau_2$-$\bar{g}$-open set in (X, τ_1, τ_2).

Theorem 3.7. Every $\tau_1\tau_2$-$\bar{g}$-open set in (X, τ_1, τ_2) is a regular generalized open set or rg-open set in both τ_1 and τ_2.

Proof. Let A be any $\tau_1\tau_2$-$\bar{g}$-open set in (X, τ_1, τ_2) and F_i be any regular closed set in (X, τ_i) contained in A for $i = 1, 2$ respectively. Since every regular closed set is a closed set, $F_i \subseteq \tau_i$-int(A) whenever $F_i \subseteq A$ and F_i is regular closed set in (X, τ_i) for $i = 1, 2$. Therefore, A is a rg-open set in both τ_1 and τ_2. □

The converse of the above theorem need not be true as seen from the following example.

Example 3.8. Let $X = \{a, b, c\}, \tau_1 = \{\phi, \{a, b\}, \{b, c\}, \{b\}, X\}$, and $\tau_2 = \{\phi, \{a, b\}, \{a, c\}, \{a\}, X\}$. Then, the set $\{c\}$ is a regular generalized open set in both τ_1 and τ_2. But, it is not a $\tau_1\tau_2$-$\bar{g}$-open.

Theorem 3.9. Every $\tau_1\tau_2$-$\bar{g}$-open set in (X, τ_1, τ_2) is a αg-open set in both τ_1 and τ_2.

Proof. Let A be any $\tau_1\tau_2$-$\bar{g}$-open set in (X, τ_1, τ_2) and F_i be any closed set in (X, τ_i) contained in A for $i = 1, 2$ respectively. Then, $F_i \subseteq \tau_i$-int(A) whenever $F_i \subseteq A$ and F_i is closed set in (X, τ_i) for $i = 1, 2$. Since τ_i-int$(A) \subseteq \tau_i$-α int(A), $F_i \subseteq \tau_i$-α int(A) whenever $F_i \subseteq A$ and F_i is closed set in (X, τ_i) for $i = 1, 2$. Therefore, A is a αg-open set in both τ_1 and τ_2. □

The converse of the above theorem need not be true as seen from the following example.

Example 3.10. Let $X = \{a, b, c\}, \tau_1 = \{\phi, \{a, b\}, X\}$, and $\tau_2 = \{\phi, \{a, c\}, X\}$. Then, the set $\{b, c\}$ is a αg-open set in both τ_1 and τ_2. But, it is not a $\tau_1\tau_2$-$\bar{g}$-open.

Theorem 3.11. Every $\tau_1\tau_2$-$\bar{g}$-open set in (X, τ_1, τ_2) is a gp-open set in both τ_1 and τ_2.

Proof. Let A be any $\tau_1\tau_2$-$\bar{g}$-open set in (X, τ_1, τ_2) and F_i be any closed set in (X, τ_i) contained in A for $i = 1, 2$ respectively. Then, $F_i \subseteq \tau_i$-int(A) whenever $F_i \subseteq A$ and F_i is closed set in (X, τ_i) for $i = 1, 2$. Since τ_i-int$(A) \subseteq \tau_i$-p int(A), $F_i \subseteq \tau_i$-p int(A) whenever $F_i \subseteq A$ and F_i is closed set in (X, τ_i) for $i = 1, 2$. Therefore, A is a gp-open set in both τ_1 and τ_2. □

The converse of the above theorem need not be true as seen from the following example.

Example 3.12. Let $X = \{a, b, c\}, \tau_1 = \{\phi, \{a, b\}, X\}$, and $\tau_2 = \{\phi, \{b\}, \{a, c\}, X\}$. Then, the set $\{b, c\}$ is a *gp*-open set in both τ_1 and τ_2. But, it is not a $\tau_1\tau_2$-$\bar{g}$-open.

Theorem 3.13. Every $\tau_1\tau_2$-$\bar{g}$-open set in (X, τ_1, τ_2) is a *gsp*-open set in both τ_1 and τ_2.

Proof. Let A be any $\tau_1\tau_2$-$\bar{g}$-open set in (X, τ_1, τ_2) and F_i be any closed set in (X, τ_i) contained in A for $i = 1, 2$ respectively. Then, $F_i \subseteq \tau_i$-int(A) whenever $F_i \subseteq A$ and F_i is closed set in (X, τ_i) for $i = 1, 2$. Since τ_i-int$(A) \subseteq \tau_i$-*sp* int(A), $F_i \subseteq \tau_i$-*sp* int(A) whenever $F_i \subseteq A$ and F_i is closed set in (X, τ_i) for $i = 1, 2$. Therefore, A is a *gsp*-open set in both τ_1 and τ_2. □

The converse of the above theorem need not be true as seen from the following example.

Example 3.14. Let $X = \{a, b, c\}, \tau_1 = \{\phi, \{a, b\}, X\}$, and $\tau_2 = \{\phi, \{b\}, \{a, c\}, X\}$. Then, the set $\{b, c\}$ is a *gsp*-open set in both τ_1 and τ_2. But, it is not a $\tau_1\tau_2$-$\bar{g}$-open.

Theorem 3.15. Every ω-open set in both τ_1 and τ_2 is a $\tau_1\tau_2$-$\bar{g}$-open set.

Proof. Let A be ω-open set in both τ_1 and τ_2 and F_i be any closed set in τ_i contained in A for $i = 1, 2$ respectively. Then, $F_i \subseteq$ int(A) whenever $F_i \subseteq A$ and F_i is a semi closed set in τ_i for $i = 1, 2$. Since every semi closed set is a closed set, $F_i \subseteq$ int(A) whenever $F_i \subseteq A$ and F_i is a closed set in τ_i for $i = 1, 2$. Hence every ω-open set in both τ_1 and τ_2 is a $\tau_1\tau_2$-$\bar{g}$-open set. □

The converse of the above theorem need not be true as seen from the following example.

Example 3.16. Let $X = \{a, b, c\}; \tau_1 = \{\phi, \{a\}, X\}$; $\tau_2 = \{\phi, \{b\}, X\}$ and $A = \{a, b\}$. Hence the set A is a $\tau_1\tau_2$-$\bar{g}$-open. But, it is not a ω-open set in τ_1 and τ_2.

Theorem 3.17. Every $r\delta$-open set in both τ_1 and τ_2 is a $\tau_1\tau_2$-$\bar{g}$-open set.

Proof. Let A be $r\delta$-open set in both τ_1 and τ_2 and F_i be any closed set in τ_i contained in A for $i = 1, 2$ respectively. Then, $A =$int$_\delta(A)$ whenever $F_i \subseteq A$ and F_i is a regular closed in τ_i for $i = 1, 2$. So, $F_i \subseteq$ int(A) whenever $F_i \subseteq A$ and F_i is a closed in τ_i for $i = 1, 2$ as every regular closed set is closed. Hence every $r\delta$-open set in τ_1 and τ_2 is a $\tau_1\tau_2$-$\bar{g}$-open set. □

The converse of the above theorem need not be true as seen from the following example.

Example 3.18. Let $X = \{a, b, c\}; \tau_1 = \{\phi, \{a\}, \{b, c\}, X\}$; $\tau_2 = \{\phi, \{c\}, \{b, c\}, X\}$ and $A = \{a, c\}$. Hence the set A is a $\tau_1\tau_2$-$\bar{g}$-open. But, it is not a $r\delta$-open set in τ_1 and τ_2.

Theorem 3.19. If A is $\tau_1\tau_2$-$\bar{g}$-open set, then A is rgb$^\sharp$-open set in both τ_1 and τ_2.

Proof. Let A be any $\tau_1\tau_2$-$\bar{g}$-open set in X such that $F_i \subseteq A$ and F_i is regular closed of τ_i for $i = 1, 2$ respectively. Hence A is g-open set in (X, τ_1) and (X, τ_2) as every regular closed set is closed, F_i is closed for $i = 1, 2$. Also, $F_i \subseteq \tau_i$-int$(A) \subseteq \tau_i$-b$^\sharp$int(A) for $i = 1, 2$. Therefore, $F_i \subseteq \tau_i$-b$^\sharp$int(A) and F_i is regular closed for $i = 1, 2$. Hence A is rgb$^\sharp$-open set in τ_1 and τ_2. □

The converse of the above theorem need not be true as seen from the following example.

Example 3.20. Let $X = \{a, b, c\}; \tau_1 = \{\phi, \{a, b\}, X\}$; $\tau_2 = \{\phi, \{b\}, \{a, c\}, X\}$ and $A = \{b, c\}$. Hence the set A is a rgb$^\sharp$-open set in both τ_1 and τ_2. But, it is not a $\tau_1\tau_2$-$\bar{g}$-open set.

Theorem 3.21. Every $\delta\hat{g}$-open set in both τ_1 and τ_2 is a $\tau_1\tau_2$-$\bar{g}$-open set.

Proof. Let A be an $\delta\hat{g}$-open set in τ_1 and τ_2 and F_i is any closed set contained in A in (X, τ_i) for $i = 1, 2$, respectively. Since every closed set is $\hat{g}$-closed and A is $\delta\hat{g}$-open set in τ_1 and τ_2, $F_i \subseteq \tau_i$-int$_\delta(A)$ for every subset A of X for $i = 1, 2$. Since $F_i \subseteq \tau_i$-int$_\delta(A) \subseteq \tau_i$-int$(A)$, $F_i \subseteq \tau_i$-int(A), and hence A is g-open set in τ_1 and τ_2. Therefore, A is $\tau_1\tau_2$-$\bar{g}$-open set. □

The converse of the above theorem need not be true as seen from the following example.

Example 3.22. Let $X = \{a, b, c\}$; $\tau_1 = \{\phi, \{b\}, \{a, c\}, X\}$; $\tau_2 = \{\phi, \{c\}, \{a, b\}, X\}$ and $A = \{b, c\}$. Hence the set A is a $\tau_1\tau_2$-$\bar{g}$-open. But, it is not a $\delta\hat{g}$-open set in τ_1 and τ_2.

Theorem 3.23. Every $\tau_1\tau_2$-$\bar{g}$-open set is strongly (gsp)*-open set in both τ_1 and τ_2.

Proof. Let A be $\tau_1\tau_2$-$\bar{g}$-open set. Then, $F_i \subseteq \tau_i$-int(A) whenever $F_i \subseteq A$ and F_i is closed in τ_i for $i = 1, 2$ respectively. Now, let $F_i \subseteq A$ and F_i be (gsp)-closed in τ_i for $i = 1, 2$ respectively. Since every closed set is a (gsp)-closed set, we have $F_i \subseteq \tau_i$-int(A) whenever $F_i \subseteq A$ and F_i is (gsp)-closed in τ_i for $i = 1, 2$. But, $F_i \subseteq \tau_i$-int$(A) \subseteq \tau_i$-int(cl(A)) whenever $F_i \subseteq A$ and F_i is (gsp)-closed in τ_i for $i = 1, 2$. Hence $F_i \subseteq \tau_i$-int(cl(A)) whenever $F_i \subseteq A$ and F_i is (gsp)-closed in (X, τ_i) for $i = 1, 2$. Therefore, A is strongly (gsp)*-open set in τ_1 and τ_2. □

The converse of the above theorem need not be true as seen from the following example.

Example 3.24. Let $X = \{a, b, c\}; \tau_1 = \{\phi, \{a\}, \{a, b\}, X\}; \tau_2 = \{\phi, \{c\}, \{b, c\}, X\}$ and $A = \{a, c\}$. Hence the set A is a strongly (gsp)*-open set in τ_1 and τ_2. But, it is not a $\tau_1\tau_2$-$\bar{g}$-open.

Theorem 3.25. If τ_1 is coarser than τ_2, then every $\tau_1\tau_2$-$\bar{g}$-open set is a $\tau_1\tau_2$-g-open set.

Proof. Let A be $\tau_1\tau_2$-$\bar{g}$-open set. Then, $F_i \subseteq \tau_i$-int(A) whenever $F_i \subseteq A$ and F_i is τ_i-closed for each $i = 1, 2$. Since τ_1 is coarser than τ_2, we have $F \subseteq \tau_2$-int(A), whenever $F \subseteq A,\ F$ is τ_1-closed. Hence A is a $\tau_1\tau_2-$g-open set.

□

Theorem 3.26. If τ_1 is coarser than τ_2, then every $\tau_1\tau_2$-$\bar{g}$-open set is a $\tau_1\tau_2$-sg-open set.

Proof. Let A be $\tau_1\tau_2$-$\bar{g}$-open set. Then, $F_i \subseteq \tau_i-$int(A) whenever $F_i \subseteq A$ and F_i is τ_i-closed for each $i = 1, 2$. Since every closed set is semi-closed, F_1 is τ_1-semi closed. Therefore, $F_1 \subseteq \tau_1$-sint(A), whenever $F_1 \subseteq A,\ F_1$ is τ_1-semi closed. Since τ_1 is coarser than τ_2, $F_1 \subseteq \tau_2$-sint(A), whenever $F_1 \subseteq A,\ F_1$ is τ_1-semi closed. Hence A is a $\tau_1\tau_2$-sg-open set.

□

Theorem 3.27. If τ_1 is coarser than τ_2, then every $\tau_1\tau_2$-$\bar{g}$-open set is a $\tau_1\tau_2$-gs-open set.

Proof. Let A be $\tau_1\tau_2$-$\bar{g}$-open set. Then, $F_i \subseteq \tau_i$-int(A) whenever $F_i \subseteq A$ and F_i is τ_i-closed for each $i = 1, 2$. Therefore, $F_1 \subseteq \tau_1$-sint(A), whenever $F_1 \subseteq A, F_1$ is τ_1-closed. Since τ_1 is coarser than τ_2,
$F_1 \subseteq \tau_2$-sint(A), whenever $F_1 \subseteq A,\ F_1$ is τ_1-closed. Hence A is a $\tau_1\tau_2-$gs-open set.

□

Theorem 3.28. If τ_1 is coarser than τ_2, then every $\tau_1\tau_2$-$\bar{g}$-open set is a $\tau_1\tau_2$-αg-open set.

Proof. Let A be $\tau_1\tau_2$-$\bar{g}$-open set. Then, $F_i \subseteq \tau_i$- int(A) whenever $F_i \subseteq A$ and F_i is τ_i-closed for each $i = 1, 2$. Therefore, $F_1 \subseteq \tau_1$-α int(A), whenever $F_1 \subseteq A$. Since τ_1 is coarser than τ_2, $F_1 \subseteq \tau_2$-α int(A), whenever $F_1 \subseteq A,\ F_1$ is τ_1-closed. Hence A is a $\tau_1\tau_2$-αg-open set.

□

Theorem 3.29. If τ_1 is coarser than τ_2, then every $\tau_1\tau_2$-$\bar{g}$-open set is a $\tau_1\tau_2$-$g\alpha$-open set.

Proof. Let A be $\tau_1\tau_2$-$\bar{g}$-open set. Then, $F_i \subseteq \tau_i$- int(A) whenever $F_i \subseteq A$ and F_i is τ_i-closed for each $i = 1, 2$. Therefore, $F_1 \subseteq \tau_1$-α int(A) whenever $F_1 \subseteq A, F_1$ is τ_1-α-closed as every closed set is α-closed. Since τ_1 is coarser than τ_2, τ_2-$F_1 \subseteq \alpha$ int(A), whenever $F_1 \subseteq A$, F_1 is τ_1-α-closed. Hence A is a $\tau_1\tau_2$-$g\alpha$-open set. □

Theorem 3.30. If τ_1 is coarser than τ_2, then every $\tau_1\tau_2$-$\bar{g}$-open set is a $\tau_1\tau_2$-$\widehat{g}$-open set.

Proof. Let A be $\tau_1\tau_2$-$\bar{g}$-open set. Then, $F_1 \subseteq \tau_1$-int(A), F_1 is τ_1-closed. Since every closed set is semi closed, $F_1 \subseteq \tau_1$-int(A), whenever $F_1 \subseteq A, F_1$ is τ_1-semi closed. Since τ_1 is coarser than τ_2, $F_1 \subseteq \tau_2$ int(A), whenever $F_1 \subseteq A$, F_1 is τ_1-semi closed. Hence A is a $\tau_1\tau_2$-$\widehat{g}$-open set. □

Definition 3.31. A function f from spaces (X, τ_1, τ_2) into (Y, σ_1, σ_2) is called $\tau_1\tau_2 - \bar{g}-$continuous if $f^{-1}(V)$ is $\tau_1\tau_2 - \bar{g}-$open set in X for each σ_i-open set V in Y.

Example 3.32. Let $X = \{a, b, c\} = Y; \tau_1 = \{\phi, \{a, b\}, X\}$; $\tau_2 = \{\phi, \{b\}, \{a, c\}, X\}; \sigma_1 = \{\phi, Y\}$ and $\sigma_2 = \{\phi, \{b\}, Y\}$. Then $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ defined by $f(a) = a$ is $\tau_1\tau_2 - \bar{g}-$continuous mapping.

Theorem 3.33. If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is $\tau_1\tau_2 - \bar{g}-$continuous and $g : (Y, \sigma_1, \sigma_2) \to (Z, \rho_1, \rho_2)$ is continuous, then $g \circ f$ is $\tau_1\tau_2 - \bar{g}-$continuous.

Proof. Let A be ρ_i- open set in Z. Since g is continuous, $g^{-1}(A)$ is σ_i-open in Y. Since f is $\tau_1\tau_2 - \bar{g}-$continuous, $f^{-1}(g^{-1}(A))$ is $\tau_1\tau_2 - \bar{g}-$open in X. Hence $g \circ f$ is $\tau_1\tau_2 - \bar{g}-$continuous. □

4 Conclusion

In this paper, $\tau_1\tau_2$-$\bar{g}$-open sets were introduced in the bitopological spaces and their properties were studied. Further, their properties were compared with some of the corresponding generalized open sets in the general topological spaces and bitopological spaces.

Competing Interests

Authors have declared that no competing interests exist.

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