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Short Communication

Degree of vertices and number of edges in a mixed graph

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In the study of a graph theory, degrees of vertices and number of edges are among the important theoretical terms that are helpful in showing discrete structures and their properties. This study attempts to present the relationship between number of edges and degrees of vertices in a mixed graph.

Key words: Graph, directed graphs, undirected graphs, mixed graph, degree of vertex, number of edges.

INTRODUCTION

Graph is a discrete structure consisting of vertices and edges that connect these vertices. To model problems and physical situations in real life, graph models with edges fully oriented (directed graph) or fully un-oriented (un-directed graph) are necessary. But sometimes, it is necessary to use mixed graph models, a graph in which some of the edges are directed while others are un-directed.

Edges and vertices are the building blocks of a graph and the structural information conveyed by graphs. The structural information conveyed by graphs is basically information with the edges of the graphs representing channels through which a service is delivered while the vertices of the graph are points/destinations where the delivery of the service originates, transmitted or ceased.

In the study of a graph theory, degrees of vertices and number of edges are among the important theoretical terms which are helpful in the study of discrete structures and their properties. The relationship between number of edges and degrees of vertices in directed and undirected graphs has been studied and presented in previous publications and many contemporary studies. This study, however, attempts to present the relationship between number of edges and degrees of vertices in a mixed graph. All the theoretical terms not defined but used in this study are presented in the work of Rosen (2007).

DEGREES OF VERTEX AND NUMBER OF EDGES

Mixed graph $G_m = (V, E, \vec{E})$ is a graph containing un-oriented E as well as oriented set of edges $\vec{E}$ (Sotskov and Tanaev, 1976). Mixed graph is used in engineering, physical science, biological science, communication technology, computer technology and other physical situations involving discrete objects where some of their edges are un-directed, while others are directed.

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Although, mixed graph models are used in diverse fields, only the following three examples will be presented here.

Degree of vertices has applications in different fields of study. For example, in telephone call graph: in a directed graph, *deg* $^-$*(v)* is the number of calls *v* received and *deg*$^+$*(v)* is the number of calls *v* made; in un directed graph , *deg(v)* is the number of calls either made or received. Again if a vertex *v* in road map graph containing one way and two way roads represents a square/ intersection, then the in degree *deg* $^-$*(v)* and out degree *deg*$^+$*(v),* respectively, represents the number of one way roads taken to and away from the square *v*; and the degree *deg(v)* is the number of two way roads that joined with *v.*

Handshaking theorem (Rosen, 2007) relates degrees of vertices and number of edges in undirected and directed graphs, respectively, by:

$$2|E| = \sum_{v\in V}\deg(v) \text{ and } |\vec{E}| = \sum_{v\in V}\deg^-(v) = \sum_{v\in V}\deg^+(v)$$

Where, V_1 and V_2 are sets of vertices of undirected graph $G_u = (V_1, E)$ and directed graphs $G_d = (V_2, \vec{E})$. Based on these, the following results are presented.

RELATIONSHIP BETWEEN DEGREE OF VERTICES AND NUMBER OF EDGES IN A MIXED GRAPH

As described above, mixed graphs are used to model problems that require a pair of task processors represented by directed and undirected edges. Thus, unlike the degree of a vertex in directed and undirected graph, the degree of vertex in a mixed graph corresponds with both directed and undirected edges, that is, a given vertex in a mixed graph executes tasks by receiving input data from the incoming directed and un-directed edges, then sends it along the desired outgoing directed and undirected edges. Thus, in the remaining part of this paper, the following definition and notation is used for the degree of vertex in mixed graph.

Definition 4.1

For a vertex *v* of a mixed graph $G_m = (V, E, \vec{E})$, the degree of a vertex *v*, denoted by *deg (v)*, is the number of undirected edges incident with *v*. Moreover, the in-degree of a vertex *v*, denoted by $\deg^-(v)$, is the number of edges with *v* as their terminal vertex and the out degree of *v*, denoted by $\deg^+(v)$, is the number of edges with *v* as their initial vertex.

There are many properties of graph with directed edges that do not depend on the direction of its edges. For example, the number of edges of directed graph and its underlying undirected graph is the same (Rosen, 2007). But the physical meanings of the vertices of these graphs might be different. For instance, if a road map graph model containing one and two way roads is treated as fully oriented or fully un-oriented graph, there might be failure in conveying the required information to the users; because the traffic weight and the service of one and two way roads may not be exactly the same. Thus, this manuscript suggests that, for due attention, the theoretical concepts and properties mixed graph from their underlying directed or undirected graphs be studied.

Corollary 4.1

Let $V_m = \{v_1, v_2, v_3, \ldots v_n\}$ be the vertex set of mixed graph $G_m = (V, E, \vec{E})$). The total number of edges $|E_m|$ in the mixed graph G_m is given by:

$$|E_m| = \frac{1}{2}\sum_{v\in V}\deg(v) + \sum_{v\in V}\deg^-(v) \quad or \quad |E_m| = \frac{1}{2}\sum_{v\in V}\deg(v) + \sum_{v\in V}\deg^+(v)$$

Proof. Let $G_d = (V, \vec{E})$ be a directed and $G_u = (V, E)$ undirected edge disjoint sub graphs of $G_m = (V, E, \vec{E})$ such that $E_m = E \cup \vec{E}$

From Handshaking theorem, we get:

$$|E| = \frac{1}{2}\sum\deg(v) \text{ and } |\vec{E}| = \sum_{v\in V}\deg^-(v) = \sum_{v\in V}\deg^+(v) \qquad (1)$$

Again using elementary counting principle, we have:

$$|E_m| = |E \cup \vec{E}| = |\vec{E}| + |E| - |E \cap \vec{E}| \qquad (2)$$

However, $G_d = (V, \vec{E})$ and $G_u = (V, E)$ are edge disjoint sub graphs. Hence, $|E \cap \vec{E}|$. Thus **(2)** is reduced to $|E_m| = |\vec{E}| + |E|$ and combining with (1), we have the following:

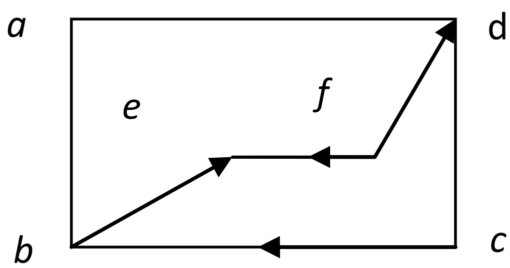

Figure 1. Graph G_m.

Table 1. Degrees of vertices of graphs G_m.

Vertex	Mixed graph Gm		
	$\deg(v)$	$\deg^-(v)$	$\deg^+(v)$
a	2	0	0
b	1	1	1
c	1	0	1
d	2	1	0
e	0	2	0
f	0	0	2

$$|E_m| = \sum_{v \in V} \deg^-(v) + \frac{1}{2}\sum_{v \in V} \deg(v) = \sum_{v \in V} \deg^+(v)$$

Example 4.1. Find the degrees of the vertices of graphs G_m (Figure 1).

Solution. Let *deg(v)* be the degree of the vertex due to the undirected edges while $deg^-(v)$ and $deg^+(v)$ are the in- and out-degrees of the vertices due to the directed edges. Then, we have the following (Table 1).

CONFLICT OF INTERESTS

The author has not declared any conflict of interests.

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