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Review

Division by zero in real and complex number arithmetic and some of its implications

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In this paper, an attempt is made to include division by zero in ordinary arithmetic. Counting or measuring is done to get the value of multiples, powers, quotients, sums and differences of zeros. Zero divided by zero is taken to be equal to one. There are many infinities and many zeroes. The biggest of all infinities cannot be imagined and the smallest zero cannot be imagined also. An attempt is made to define a mathematical structure from the set of real numbers, zeros and infinities. Some fallacies have been resolved, implications of allowing of division by zero on the quadratic equation where $a = 0$ is discussed and Newton's law of gravity and light deflection by gravity is also discussed.

Key words: Division by zero, number, Hilbert hotel, zero, infinity, -0, negative zero, nothing, Euler number.

INTRODUCTION

In most branches of mathematics, division by zero is not allowed. The result of division by zero in ordinary arithmetic is undefined (Wikipedia, 2020b). If allowed it results in contradictions. The existence of infinity is doubted by some, for example Aristotle thought actual infinity was impossible (Wikipedia, 2020a). In this paper it is argued that zero should be treated like any other number. This means there will be multiples, quotients and powers of zero. This however poses many challenges because real or complex numbers with a zero having this property cannot form a field. However, certain properties are possessed by the numbers that they can form a structure. With such a structure many fallacies about division by zero can be proven wrong.

LITERATURE REVIEW

Division by zero has been studied over a long period of time. However, despite such long period of study by various mathematicians there is still no consensus on the value of 0/0 and the value to be obtained from the division of any real number by zero. It is common to describe 0/0 as indeterminate and 1/0 as impossible. Brahmagupta (1966) stated that 0/0 = 0. Mwangi (2018) concluded that 0/0 is equal to one and that division of a non-zero number by zero is indefinite. In this paper division of zero by zero is equal to one. Ufuoma (2019) distinguishes between an intuitive zero and a numerical zero and defines 1/0 to be $\frac{1}{(-1)(-2)(-3)\ldots}$. In this paper 1/0 is equal to $\frac{1}{1+1+1+\cdots}$. Barukčić and Barukčić (2016) use Einstein's theory of relativity to deduce that 0/0 must be 1.

METHODOLOGY

An attempt is going to be made to develop some axioms for the arithmetic involving division by zero. Then with those axioms some known results will be proven and

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some common fallacies about the division by zero will be proven wrong.

Zero and infinite

If someone has a zero, how many zeros does he/she have? If someone has two zeros, how many zeros does he/she have? Suppose there are two numbers x ∈ R and y ∈ R, then:

$$0 + 0 = 2 \bullet 0 \quad (1)$$

$$0 \div 0 = 1 \quad (2)$$

$$0 - 0 = 0(1-1) = 0 \times 0 = 0^2 \quad (3)$$

$$\frac{x}{0} + \frac{y}{0} = \frac{x+y}{0} \quad (4)$$

$$\frac{1}{0} - \frac{1}{0} = \frac{0}{0} \quad (5)$$

$$\frac{x}{0} \times 0 = x \quad (6)$$

$$0 + x = 0 + x \quad (7)$$

$$0 \times x = 0x \quad (8)$$

$$0 \div x = \frac{0}{x} \quad (9)$$

$$x \div 0 = \frac{x}{0} \quad (10)$$

$$\mathrm{x} - \mathrm{y} = (\mathrm{x} - \mathrm{y}) + 0\mathrm{y} \quad (11)$$

Assuming

$$\mathrm{y} < \mathrm{x}$$

and

$$(\mathrm{x} - \mathrm{y})$$

is the ordinary difference

When we are counting/measuring, here is what we will be counting/measuring

$$\ldots, 0^4, 0^3, 0^2, 0^1, 0^0, 0^{-1}, 0^{-2}, 0^{-3}, 0^{-4}, \ldots 0^{\frac{a}{b}}$$

where a and b are real numbers, 0^d where d is any number. Thus we have powers of zeros and powers of infinity. Of all these mathematical objects $\frac{0}{0}$ is the most commonly counted/measured.

Field axioms

From Wikipedia, "Associativity of addition and multiplication:

$$a + (\mathrm{b} + \mathrm{c}) = (a + \mathrm{b}) + \mathrm{c}$$

and

$$a \cdot (\mathrm{b} \cdot \mathrm{c}) = (a \cdot \mathrm{b}) \cdot \mathrm{c}$$

Commutativity of addition and multiplication:

$$a + \mathrm{b} = \mathrm{b} + a$$

and

$$a \cdot \mathrm{b} = \mathrm{b} \cdot a$$

Additive and multiplicative identity: there exist two different elements 0 and 1 in F such that

$$a + 0 = a$$

and

$$a \cdot 1 = a$$

Additive inverses: for every a in F, there exists an element in F, denoted $-a$, called the additive inverse of a, such that

$$a + (-a) = 0$$

Multiplicative inverses: for every $a \neq 0$ in F, there exists an element in F, denoted by a

$$a^{-1} \text{ or } \frac{1}{a}$$

called the multiplicative inverse of a, such that

$$a \cdot a^{-1} = 1$$

Distributivity of multiplication over addition:

$$a \cdot (\mathrm{b} + \mathrm{c}) = (a \cdot \mathrm{b}) + (a \cdot \mathrm{c})$$

This may be summarized by saying: a field has two operations, called addition and multiplication; it is an abelian group under addition with 0 as the additive identity; the nonzero elements are an abelian group under multiplication with 1 as the multiplicative identity; and multiplication distributes over addition" (Wikipedia, 2020c).

Additive identity

There is a problem if an attempt is made to form groups with addition.

Ordinary zero can no longer be addition identity. The inverse of x is usually taken to be -x. Adding $\mathrm{x}+-\mathrm{x}$ gives $0\mathrm{x}$ and not 0, the usual identity. To get a smaller number subtract $0\mathrm{x}-0\mathrm{x}=x0^2$.

Subtracting each result from itself on and on will result in a number very small. Adding all the numbers to be subtracted will give the new inverse.

A new inverse for each number x can be taken as:

$$-\Delta\mathrm{x}$$

where Δx means

$$x+x0+x0^2+x0^3+x^4 0^4 - \ldots$$

The identity will be $0-\Delta 0$. This has been introduced to allow the solving of equations. There is actually no identity. $0-\Delta 0$ is only a poor but useful approximate of the addition identity. Declaring this number as an identity should be delayed as much as possible when doing calculations.

Division

Division of a number by itself

$$\frac{a}{a}=a^{1-1}$$

$$\frac{a}{a}=a^{0}$$

which is not necessarily equal to one. Taking p to be the traditional $\frac{b}{a}$ we have

$$\frac{b}{a}=\frac{pa}{a}$$

$$\frac{b}{a}=pa^{0} \tag{12}$$

The multiplicative inverse is $a^{-\Delta 1}$ and the identity remains one. Care should be taken when using this identity because it is not a true identity. It should only be used for convenience. The best place to reduce $a^{(\mathrm{x}-\Delta\mathrm{x})}$ to $a^{0-\Delta 0}$ and finally to a is in the final answer.

After these changes have been done.

New field axioms

Associativity of addition and multiplication.

Some manipulation needs to be done when changing the parenthesis without changing the value of an expression. For example, for any positive numbers a and b

$$a+(b+c)=(a+b)+c$$
$$a+(b+-c)=(a+b)+-c$$
$$a+-(b+c)=(a+-b)+-c$$
$$a+-(b+-c)=(-b+a)-c$$

$$-a+(b+c)=(-a+b)+c$$
$$-a+(b+-c)=(-a+b)+-c$$
$$-a+-(b+c)=(-a+-b)+-c$$
$$-a+-(b+-c)=(-a+-b)-c$$

A negative number that was being added to a positive number that is coming after it in an expression should never be added to a positive number that is coming before it without making necessary adjustments.

$$a+(-b+c)\neq(a+-b)+c$$

but

$$a+(-b+c)=(a+-b)+c-0b-0b\Delta$$

Since –b was being added to c which is coming after it in the expression, it cannot be added to a which is coming before it in the expression.

However, it is much easier to use the relation:

$$a+(-b+c)=(-b+a)+c$$

Also for any numbers a, b and c

$$a\cdot(b\cdot c)=(a\cdot b)\cdot c \tag{13}$$

Commutativity of addition and multiplication

For any positive numbers a and b

$$a+b=b+a$$

For $a>b$

$$a-b=-b+a+0b\Delta$$
$$-a+b=b-a-0b\Delta$$

and for any numbers a and b whose absolute values are both greater than or equal to one or both less than one

$$a\cdot b=b\cdot a.$$

Otherwise both numbers should be expressed in index form with a common base and the commutative property for addition should then be used.

Additive and multiplicative identity

There exist two different elements 0 - ∆0 and 1 in F such that

$$a + 0 - \Delta 0 = a \tag{14}$$

and

$$a \cdot 1 = a$$

These are only approximations.

Additive inverse

For every a in F, there exists an element in F, denoted $-a\Delta$, called the additive inverse of a, such that

$$a + (-a\Delta) = 0 - \Delta 0 \tag{15}$$

approximately.

Multiplicative inverse

For every a in F, there exists an element in F, denoted by $a^{-\Delta 1}$ or $\frac{1}{a^{-\Delta 1}}$, called the multiplicative inverse of a, such that $a \cdot a^{-\Delta 1}$ = 1 approximately.

Distributivity of multiplication over addition:

$$a \cdot (b + c) = (a \cdot b) + (a \cdot c)$$

This may be summarized by saying: a field has two operations, called addition and multiplication; it is an abelian group under addition with 0-∆0 as the additive identity; and an abelian group under multiplication with 1 as the multiplicative identity and multiplication distributes over addition.

Infinity

The simplest infinity, that is $\frac{1}{0}$, is considered first. It tells how many zeros are in one. Infinity means without limit. There are many infinities, some greater than others. They have the same property, being unreachable.

Zero

Since 0 ×$\frac{1}{0}$ = 1, 0 could be considered something. If any symbol or name is given to nothing, then that symbol × $\frac{1}{that\ symbol}$ = 1. Nothing × $\frac{1}{Nothing}$ = 1. In an attempt to get just closer to the number the iteration:

$x_{n+1} = x_n - x_n$ can be used.

This shall be written

$$x - x - \Delta x.$$

The value $0a$ will be used as the difference a-a.

From Wikipedia, "Aristotle postulated that an actual infinity was impossible, because if it were possible, then something would have attained infinite magnitude, and would be "bigger than the heavens." However, he said, mathematics relating to infinity was not deprived of its applicability by this impossibility, because mathematicians did not need the infinite for their theorems, just a finite, arbitrarily large magnitude" (Wikipedia, 2020a).

Actual infinity implies actual nothing which is needed for example when solving equations. Division, subtraction, multiplication and subtraction cannot map a set of finite numbers to actual infinity. Potential infinity goes a long way in helping mathematicians in their theorems by approximating actual infinity.

To solve equations, we therefore do not necessarily need nothing if we need an answer that is just accurate enough.

$$2x+1=7$$

$$2x+1-1-\Delta=7-1-\Delta$$

$$2x=6+1-1-\Delta$$

What happened to 1 which was in front of 2x? What has taken its place? There is nothing between 2x and the equal sign. It is not a true nothing however there is 0.5-∆0.5 which is smaller than 0-∆0. When nothing has been written down it does not imply perfect nothing, it simply means that the value there is so small that even if we were to divide that number by the zero raised to the power of a number bigger than all the numbers we are working with in this problem, we will still get an insignificantly small number.

The degree of nothing will be denoted by the coefficient of zero and its index also. This means 1 can be taken as nothing in some cases and so is $\frac{1}{0}$. The degree of rounding depends on context.

Division by 1 + b0

To divide a number by a sum or difference of different levels of zero an approximation is used. The choice of the number of terms to be taken depends on the required accuracy of the answer. Using the binomial expansion and leaving out higher powers of zero

$$\frac{1}{(2 + 3\bullet 0)} = 1 \times 0.5(1 - \frac{3\bullet 0}{2})$$

Roots of 1 + b0

To find a root of a sum or difference of different objects an approximation is used. The choice of the number of terms to be taken depends on the required accuracy of the answer.

$$(2+3\bullet 0)^{\frac{1}{2}} = 1 \times 2^{\frac{1}{2}}\ (1+\frac{3\bullet 0}{4})^{\frac{1}{2}}$$

Euler number

Euler number is given by

$$(1+0)^{\frac{1}{0}} = 1^{\frac{1}{0}} + (\frac{1}{0})\bullet 1^{\frac{1}{0}-1}\bullet 0 + \frac{(\frac{1}{0})(\frac{1}{0}-1)\bullet 1^{\frac{1}{0}-2}\bullet 0^2}{2} + \frac{(\frac{1}{0})(\frac{1}{0}-1)\bullet 1^{\frac{1}{0}-3}\bullet(\frac{1}{0}-2)0^3}{3!} + \ldots \qquad 16)$$

Since $1^{\frac{1}{0}} = 1$

$$(1+0)^{\frac{1}{0}} = 1(1+(\frac{1}{0})0 + \frac{(\frac{1}{0})(\frac{1}{0}-1)0^2}{2} + \frac{(\frac{1}{0})(\frac{1}{0}-1)(\frac{1}{0}-2)0^3}{3!} + \ldots)$$

Ignoring finite number multiples of 0 or higher powers, that is, rounding off:

$$(1+0)^{\frac{1}{0}} = 1+1+\frac{1}{2!}+\frac{1}{3!}+\ldots$$

Thus

$$e = (1+0)^{\frac{1}{0}}$$

$$e^0 = 1+0$$

$$e^x = e^{\frac{0\bullet x}{0}}$$

$$e^x = (e^0)^{\frac{x}{0}}$$

$$e^x = (1+0)^{\frac{x}{0}}$$

$$e^x = 0^x + (\frac{x}{0})(0^{x-1\bullet 0}0) + \frac{(\frac{x}{0})(\frac{x}{0}-2\bullet 0)0^{x-2\bullet 0}\ 0^2)}{2!} + \ldots)$$

For a really small number

$$(1+0-\Delta 0)^{\frac{1}{0-\Delta 0}} = 1^{\frac{1}{0-\Delta 0}}(1+(\frac{1}{0-\Delta 0})(0-\Delta 0) + \frac{(\frac{1}{0-\Delta 0})(\frac{1}{0-\Delta 0}-1)(0-\Delta 0)^2}{2} + \frac{(\frac{1}{0-\Delta 0})(\frac{1}{0-\Delta 0}-1)(\frac{1}{0-\Delta 0}-2)(0-\Delta 0)^3}{3!} + \ldots)$$

Ignoring finite number multiples of (0-Δ0) or higher powers that are, rounding off

$$(1+(0-\Delta 0))^{\frac{1}{0-\Delta 0}} = 1^{0-\Delta 0}(1+1+\frac{1}{2!}+\frac{1}{3!}+\ldots)$$

Thus e is defined to be

$$e = (1+(0-\Delta 0))^{\frac{1}{0-\Delta 0}}$$

$$e^{(0-\Delta 0)} = 1+(0-\Delta 0);$$

$$e^x = e^{\frac{(0-\Delta 0)\bullet x}{0-\Delta 0}}$$

$$e^x = (E^{(0-\Delta 0)})^{\frac{x}{0-\Delta 0}}$$

$$e^x = (1+(0-\Delta 0))^{\frac{x}{0-\Delta 0}}$$

Any number smaller than or equal to the product of zero and a finite number can be substituted for $0-\Delta 0$

Numbers raised to the power of infinity

$$\frac{1}{1-x} = 1+x+x^2+x^3+x^4+\ldots x^{\frac{1}{0}}$$

$$\frac{1}{2} = 1-1+1-1+1-1+\ldots$$

up to infinity

$$\frac{1}{2} = 0+0+0+\ldots$$

up to half infinity

$$\frac{1}{2} = 0 \times \frac{1}{2\bullet 0}$$

$$\frac{1}{2} = \frac{1}{2}$$

$$\frac{1}{0^{\frac{1}{0}-1}}$$

1 to the power of infinity. It may be assumed that

$$1^{\frac{1}{0}} = (0^0)^{\frac{1}{0}}$$

$$= 0^1$$

$$= 0$$

But $1 = 0^0$ is only approximately true

A better approximation is $1 = 0^{0-\Delta 0}$. Therefore 1 raised to the power of infinity is 1.

The relationship between zeros and ones

$0^0 = 1$ and $0^1 = 0$

Finite numbers greater than 1 are equal to zero raised to the power of a finite number multiple of zero.

Implications of division by zero

$0 \neq -0$

There are more than infinite numbers between 0 and -0. The square root of a sum of ones and zeros (x + 0•y) is

Quadratic equation where a = 0:

$$ax^2 + bx + c = 0$$

$$a(ax^2 + \frac{bx}{a}) + c = 0$$

$$a(x^2 + \frac{bx}{a} + (\frac{b}{2a})^2 - (\frac{b}{2a})^2 \quad - 0(\frac{b}{2a})^2) + \ c = 0$$

$$a(x + \frac{b}{2a})^2 + 0a(\frac{b}{2a})^2 \quad - 0a(\frac{b}{2a})^2 + c = 0$$

$$a(x + \frac{b}{2a})^2 + 0^2 \bullet a\frac{b}{2a})^2 \ - 0a(\frac{b}{2a})^2 \ + 0c \ = \ 0 - 0a(\frac{b}{2a})^2 \ - a(\frac{b}{2a})^2 + c$$

$$a(x+\frac{b}{2a})^2+0^2\bullet a(\frac{b}{2a})^2-\Delta 0^2\bullet a(\frac{b}{2a})^2-0a(\frac{b}{2a})^2+\Delta 0a(\frac{b}{2a})^2+0c-\Delta 0c=0-0a(\frac{b}{2a})^2-a(\frac{b}{2a})^2+c-\Delta 0^2\bullet a(\frac{b}{2a})^2+\Delta 0a(\frac{b}{2a})^2 - \Delta c$$

$$a(x + \frac{b}{2a})^2 \ = \ 0 - 0a\,(\frac{b}{2a})^2 \ \ - a(\frac{b}{2a})^2 \ + c - \Delta 0^2 \bullet a\,(\frac{b}{2a})^2 + \Delta 0a\,(\frac{b}{2a})^2 \ - \Delta 0c$$

$$(x + \frac{b}{2a}) = \sqrt{(\ \frac{0}{a} - 0(\frac{b}{2a}))^2 \ + (\frac{b}{2a})^2 \ + \frac{c}{a} - \Delta 0^2(\frac{b}{2a})^2 + \Delta 0\ (\frac{b}{2a})^2 - \Delta 0\frac{c}{a})}$$

$$(x + \frac{b}{2a}) = \frac{\sqrt{(\ \ 0\bullet 4a - 0b^2 \ + b^2 + 4ac - \Delta 0^2\ b^2 + \Delta 0\bullet 2b^2 - \Delta 0\bullet 4ac\ )}}{2a}$$

$$x = \frac{\sqrt{(\ \ 0\bullet 4a - 0\ b^2 \ + \ b^2 + 4ac - \Delta 0^2 b^2 + \Delta 0\ b^2 - \Delta 0\bullet 4ac\ )}}{2a} - \frac{b}{2a} - \Delta\frac{b}{2a}$$

$$x = \frac{(-b - \Delta b \pm \sqrt{(\ \ 0\bullet 4a - 0b^2 \ + \ b^2 + 4ac - \Delta 0^2\ b^2 + \Delta 0b^2 - \Delta 0\bullet 4ac\ ))}}{2a}$$

approximated by using the first two terms of the binomial expansion which are $\sqrt{x}(1+\frac{0\bullet y}{2x})$. Since the denominator is equal to zero, the highest power of 0 that will give a multiple of one is one. All terms with zero raised, to the power 2 or more will be dropped. Therefore Δ will be replaced by 0. 0•4a is to be dropped because it is a multiple of zero always.

$$x = \frac{(-b - 0b \pm \sqrt{(\ b^2 - 4ac\ )})}{2a}$$

$$x = \frac{(-b - 0b \pm (b\,(1 - \frac{4ac}{2\bullet b^2}\,))}{2a}$$

With these formulas both quadratic equations with a = 0 and those with $a \neq 0$ can be solved. The last formula simplifies work when a=0. If $a \neq 0$ then the -0b in the final answer will be a multiple of zero so if the answer is to be correct to a finite decimal places, then it will always be dropped. The same applies to 2a. Thus for $a \neq 0$ this formula is the ordinary quadratic formula.

Example

Solve the equation $0x^2 + 2x + 3 = 0$

$$x = \frac{(-2 - 0\bullet 2 \pm (2\,(1 - 4\bullet 0\bullet \frac{3}{2\bullet 2^2}))}{2\bullet 0}$$

$$x = \frac{-2 - 0\bullet 2 + 2\bullet 1 - 0\bullet 3}{2\bullet 0}$$

$$x = \frac{-2 + 2 - 0\bullet 2 - 3}{2}$$

$$x = \frac{2\bullet 0 - 0\bullet 2 - 3}{2}$$

$$x = \frac{2\bullet 0^2 - 3}{2}$$

$$x = \frac{-3}{2}$$

Integration of $\frac{1}{x}$

$$\int \frac{1}{x}\,dx = \frac{x^{-1+1}}{-1+1}$$

$$= \frac{x^0}{0}$$

Since

$$x = e^{lnx}$$

$$\int \frac{1}{x}\,dx = \frac{e^{0\ln x}}{0}$$

$$= \frac{(e^0)^{lnx}}{0}$$

$$= \frac{(1+0)^{lnx}}{0}$$

$$= \frac{1}{0}\bullet \frac{(1 + \ln x\bullet 0 + \ln x(\ln x - 1)\bullet 0^2}{2!} + \frac{\ln x(\ln x - 1)(\ln x - 2)\bullet 0^3}{3!} + \ldots$$

Ignoring finite number multiples of zero to the power one or higher

$$\int \frac{1}{x}\,dx = \frac{1}{0} + lnx$$

This is the integral of $\frac{1}{x}$

Differentiation

An attempt is made to find the derivative at a given point, that is,

$$\frac{dy}{dx} = \frac{(f(x) - f(x)}{x - x}$$

The results show that the gradient at very small interval is not the gradient mathematicians are used to.

$$\frac{d2x}{dx} = \frac{2x - 2x}{x - x}$$

$$= \frac{0\bullet 2x}{0\bullet x}$$

$$= 2$$

$$\frac{dx^2}{dx} = \frac{x^2 - x^2}{x - x}$$

$$= \frac{0x^2}{0x}$$

$$= x$$

The derivative of $\sin(x)$ is now $\frac{sinx}{x}$ which may not be satisfactory. The results show that the gradient at a point is not the gradient mathematicians are used to.
The derivative of a function is given by using x and some value very close to it which is x+0.

$$\frac{df(x)}{dx} = \frac{f(x+0) - f(x)}{x + 0 - x}$$

$$\frac{d2x}{dx} = \frac{2(x+0) - 2x}{0(x+1)}$$

$$= \frac{2(x + 0 - x)}{0(x+1)}$$

$$= \frac{2\bullet 0(x+1)}{0(x+1)}$$

$$= 2$$

$$\frac{dx^2}{dx} = \frac{(x+0)^2 - x^2}{x - x + 0}$$

$$= \frac{x^2 + 2\bullet 0x + 0^2 - x^2}{0(x+1)}$$

$$= \frac{0x^2 + 0\bullet 2x + 0^2}{0(x+1)}$$

$$= \frac{x^2 + 2x + 0}{x+1}$$

$$= \frac{x^2 + 2x}{x+1}$$

The results show that the gradient at a very small interval is not the gradient mathematicians are used to.

The derivative of f(x) now using x and x+1

$$\frac{df(x)}{dx}=(f(x+1)-f(x))$$

$$=2(x+1)-2x$$

$$=2x+2-2x$$

$$=0\bullet 2x+2$$

$$y=x^2$$
$$\frac{dy}{dx}=(x+1)^2-x^2$$

$$=0x^2+2x+1$$

$$=2x+1$$

The derivative using x and x+Δx

$$\frac{df(x)}{dx}=\frac{f(x+\Delta x0x)-f(x)}{x+\Delta 0x-x}$$

$$f(x)=2x+1$$

$$\frac{df(x)}{dx}=\frac{2(x+\Delta x)-2x}{x-\Delta x}$$

$$=\frac{2x+2\Delta 0x-2x}{x-\Delta x0x}$$

$$=\frac{2(0x-\Delta 0x)}{0x-\Delta 0x}$$

$$=2$$

$$f(x)=x^2$$

$$\frac{df(x)}{dx}=\frac{(x+\Delta 0x)^2-x^2}{0x-\Delta 0x}$$

$$=\frac{0x^2+2\Delta 0x+(\Delta 0x)^2}{0x-\Delta 0x}$$

$$=\frac{x+2\Delta x+(\Delta 0)^2}{1-\Delta}$$

$$=\frac{3x}{1-\Delta}$$

The results show that the gradient at a very small interval is not the gradient mathematicians are used to. The derivative of a function seems to be always changing as the length of chord decreases.

The change in x is now $0x+0^{0.5}$

$$f(x)=2x+1$$

$$\frac{df(x)}{dx}=\frac{2(x+0^{0.5})+1-2x-1}{0x+0^{0.5}}$$
$$=\frac{0\bullet 2x+0\bullet 1+2\bullet 0^{0.5}}{0x+0^{0.5}}$$

$$=(0\bullet 2x+0\bullet 1+2\bullet 0^{0.5})0^{-0.5}(1-x0^{0.5}+\ldots)$$

$$=2$$

$$f(x)=x^2$$
$$\frac{df(x)}{dx}=\frac{(x+0^{0.5})^2-x^2}{0x+0^{0.5}}$$
$$=\frac{x^2+2x\bullet 0^{0.5}+0-x^2}{0x+0^{0.5}}$$
$$=(0x^2+2x0^{0.5}+0)0^{-0.5}(1-x0^{0.5}+\ldots)$$

$$=2x$$

$$f(x)=x^n$$
$$\frac{df(x)}{dx}=\frac{(x+0^{0.5})^n-x^n}{0x+0^{0.5}}$$
$$=(x^n+n(x^{n-1})0^{0.5}+\frac{n(n-1)x^{n-2}0}{2!}+\ldots-x^n)0^{-0.5}(1-x0^{0.5}+\ldots)$$

$$=nx^{n-1}$$
$$f(x)=\sin x$$

$$\frac{df(x)}{dx}=\frac{\sin(x+0^{0.5})-\sin x}{0x+0^{0.5}}$$
$$=\frac{\sin x\cos 0^{0.5}+\cos x\sin 0^{0.5}-\sin x}{0x+0^{0.5}}$$
$$(\cos 0+i\sin 0)^{0^{-0.5}}=1+i0(0^{-0.5})-\frac{0^{-0.5}(-0^{-0.5}-1)0^2}{2!}+\ldots$$

$$=\frac{\sin x+0\bullet 0^{-0.5}\cos x-\sin x}{0x+0^{0.5}}$$

$$=(0\sin x+0^{0.5}\cos x)0^{-0.5}(1-x0^{0.5}+\ldots)$$

$$=\cos x$$

With the last formula differentiation will yield familiar results. A function whose derivative is itself may be in the form a^x. From first principles:

$$\frac{dy}{dx}=\frac{a^{x+0}-a^x}{0x+0}$$

Since the derivative is equal to the function:

$$a^x=\frac{a^{x+0}-a^x}{0(x+1)}$$

$$a^x = \frac{a^x(a^0 - 1)}{0(x+1)}$$

For the two sides to be equal

$$a^0 - 1 = 0(x+1)$$

$$a^0 + 0 = 0 + 1 + 0x$$

$$a^0 = 0 + 1 - \Delta 0x + 0x$$

$$a^0 = 1 + 0x$$

$$f(x) = a^x$$

$$\frac{df(x)}{dx} = \frac{a^{x+0^{0.5}} - a^x}{0x + 0^{0.5}}$$

$$a^x = \frac{a^x(a^{0^{0.5}} - 1}{0x + 0^{0.5}}$$

$$0x + 0^{0.5} = a^{0^{0.5}} - 1$$

$$0x + 0^{0.5} + 1 - \Delta 0 = a^{0^{0.5}}$$

$$a = (1 + 0^1 + 0^{0.5} + 0x)^{0^{-0.5}}$$

$$a = 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \dots$$

This is the e expected but for finite values of x or less.

The formulas for differentials can only be used without the possibility of getting wrong results for finite numbers only due to the rounding off that has been done.

Sums to infinity

Sum of all natural numbers

$$\frac{1}{(1-1)^2} = 1 + 2 + 3 + \dots + \frac{1}{0}$$

$$\frac{1}{0^2} = 1 + 2 + 3 + \dots + \frac{1}{0}$$

The series for $\frac{1}{1-x}$ and $\frac{1}{(x-1)^2}$ can be used to find the value of certain series.

Newton's law of gravity when the mass of one object is zero

Newton's universal law of gravitation, when division by zero has been allowed, predicts that massless particles are attracted by gravity. The massless particles accelerate at the same rate as any other particle.

$$F = \frac{G \bullet m \bullet 0}{r^2}$$

$$a = \frac{G \bullet m \bullet 0}{0 \bullet r^2}$$

$$a = \frac{G \bullet m}{r^2}$$

Some fallacies and results from other attempts to divide by zero revisited

Hilbert hotel

In the Hilbert's hotel a room is created by moving a guest from one room to the next displacing the incumbent into the next room (Sarazola, 2017). This process goes on forever since there is no last room. At any moment a guest in a finite position is shifting. At any moment one guest is without a room. The rooms could not accommodate a greater number of visitors than its capacity. Making guests take turns to be without a room does not create an extra room.

Factorization of difference of two squares

Ufuoma (2019) distinguishes between numerical zero and intuitive zero and concludes that $\frac{x^2-1}{x-1}$ when $x = 1$ is equal to 2. The following shows that 2 may not be the correct answer:

$$\frac{x^2 - 1}{x - 1} = \frac{(x-1)(x+1) - \Delta 0x}{x - 1}$$

When x=1

$$\frac{1^2 - 1}{1 - 1} = \frac{(1-1)(1+1) - \Delta 0 \bullet 1}{1 - 1}$$

$$\frac{0}{0} = \frac{(0)(2) - \Delta 0 \bullet 1}{0}$$

$$\frac{0}{0} = \frac{2 \bullet 0 - \Delta 0 \bullet 1}{0}$$

$$\frac{0}{0} = \frac{1 \bullet 0 + 1 \bullet 0 - \Delta 0 \bullet 1}{0}$$

$$\frac{0}{0} = \frac{1 \bullet 0 + \Delta\Delta 0 \bullet 1}{0}$$

$$\frac{0}{0} = \frac{1 \bullet 0}{0} = 1$$

Ambiguity of value

Since multiplying by zero results in unique values, there is no ambiguity of value.

Fallacy of equal numbers

From Gilreath (2016), “Suppose that:

$$a = b + c$$

where a, b, and c are positive numbers. Then in as much as a is equal to b plus some number, a is greater than b. Multiply both sides by a - b. Then

$$a0 - ab = ab + ac - b0 - bc.$$

Subtract ac from both sides:

$$a0 - ab - ac = ab + ac - b0 - bc - ac.$$

Simplify the equation:

$$a0 - ab - ac = ab - b0 - bc.$$

Factor:

$$a(a - b - c) = b(a - b - c)$$

Divide both sides by a - b - c. Then

$$a = b$$

Thus a, which was originally assumed to be greater than b, has been shown to be equal to b.”
These manipulations are redone as follows:

Suppose that:

$$a = b + c$$

where a, b, and c are positive numbers. Then inasmuch as a is equal to b plus some number, a is greater than b.

Multiply both sides by a - b. Then,

$$a0 - ab = ab + ac - b0 - bc.$$

Subtract ac from both sides:

$$a0 - ab - ac = ab + ac - b0 - bc - ac.$$

Simplify the equation:

$$a0 - ab - ac = ab - b0 - bc + 0ac.$$

Factor:

$$(a - b - c) = (a - b - c) + 0ac$$

Divide both sides by a - b - c. Then

$$a = b + \frac{0ac}{a - b - c}$$

Thus a, which was originally assumed to be greater than b, has been shown to be still greater than b for the numerical example in Gilreath (2016) the problem is solved as follows:

$$3 = 2 + 1$$

Multiply both sides by 3-2

$$3(3 - 2) = (2 + 1)(3 - 2)$$

$$3 * 3 - 3 * 2 = 2 * 3 - 2 * 2 + 1 * 3 - 1 * 2$$

Subtract 3*1 from both sides

$$3 * 3 - 3 * 2 - 3 * 1$$
$$= 2 * 3 - 2 * 2 + 1 * 3 - 1 * 2 - 3 * 1$$

Simplify

$$3 * 3 - 3 * 2 - 3 * 1$$
$$= 2 * 3 - 2 * 2 - 1 * 2 + 0 * 3 * 1$$

Factor

$$3(3 - 2 - 1) = 2(3 - 2 - 2) + 0 * 3 * 1$$

Divide both sides by 3-2-1

$$3 = 2 + \frac{0 * 3 * 1}{0 * 3}$$

$$3 = 2 + 1 = 3$$

RESULTS

Defining numbers to be multiplied and powers of zero we have the following axioms.

Associativity of addition and multiplication

For any positive numbers a and b

$$a + (b + c) = (a + b) + c$$
$$a + (b + -c) = (a + b) + -c$$
$$a + -(b + c) = (a + -b) + -c$$
$$a + -(b + -c) = (-b + a) - c$$

$$-a+(b+c)=(-a+b)+c$$
$$-a+(b+-c)=(-a+b)+-c$$
$$-a+-(b+c)=(-a+-b)+-c$$
$$-a+-(b+-c)=(-a+-b)-c$$

A negative number that was being added to a positive number that is coming after it in an expression should not ever be added to a positive number that is coming before it without making necessary adjustments.

$$a+(-b+c)\neq(a+-b)+c$$

but

$$a+(-b+c)=(a+-b)+c-0b-0b\Delta$$

Since –b was being added to c which is coming after it in the expression, it cannot be added to a the a coming before it in the expression. However, it is much easier to use the relation:

$$a+(-b+c)=(-b+a)+c$$

Also for any numbers a, b and c

$$a\cdot(b\cdot c)=(a\cdot b)\cdot c$$

Commutativity of addition and multiplication

For any positive numbers a and b

$$a+b=b+a$$

For $a>b$

$$a-b=-b+a+0b\Delta+0b$$

$$-a+b=b-a-0b\Delta-0b$$

and for any numbers a and b whose absolute values are both greater than or equal to one or both less than one

$$a\cdot b=b\cdot a.$$

Otherwise both numbers should be expressed in index form with a common base and the commutative property for addition should then be used.

Additive and multiplicative identity

There exist two different elements 0 - Δ0 and 1 in F such that

$$a+0-\Delta 0=a$$

and

$$a\cdot 1=a$$

These are only approximations.

Additive inverses

For every a in F, there exists an element in F, denoted $-a\Delta$, called the additive inverse of a, such that $a+(-a\Delta)=0-\Delta 0$ approximately.

Multiplicative inverses

For every a in F, there exists an element in F, denoted by $a^{-1-\Delta 0}$ or $\frac{1}{a^{1+\Delta 0}}$, called the multiplicative inverse of a, such that$a\cdot a^{-1-\Delta 0}$ = 1 approximately.

Distributivity of multiplication over addition

$$a\cdot(b+c)=(a\cdot b)+(a\cdot c)$$

This may be summarized by saying: a field has two operations, called addition and multiplication; it is an abelian group under addition with 0-Δ0 as the additive identity; and an abelian group under multiplication with 1 as the multiplicative identity; and multiplication distributes over addition.

DISCUSSION

We cannot freely use division by zero. Accuracy in computations is always limited. Any attempt to give the difference 1-1 or a symbol will result in a paper like this. The symbol 0 will be replaced by the new symbol if x-x is taken to be perfect nothing. If 1-1 is taken to be 0 then the value of the perfect nothing will never be reached by mathematical operations other than rounding off.

Conclusion

Division by zero is possible and if done well will not lead to any paradoxes. Division by zero of any number is very possible although calculations will become longer and there is a limit to the degree of zero that any calculation may reach.

CONFLICT OF INTERESTS

The authors have not declared any conflict of interests.

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