Development of a Competition Model for *Streptococcus mutans* and *Streptococcus sanguis* in Mixed Culture: A Mathematical Analysis

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Authors' contributions

This work was carried out in collaboration between both authors. Author MK designed the study, carried out the model analysis, wrote the draft and numerical simulation of the manuscript. Author MSH managed literature searches, species selection of the model and biological explanation of the results on all sections. Both authors read and approved the final manuscript.

Original Research Article

ABSTRACT

A competitive mathematical model for the growth of *Streptococcus mutans* and *Streptococcus sanguis* is developed in this study. The main goal of this paper is to investigate the roles of two different growth laws: The logistic growth and the Smith growth to determine the competition outcome between two organisms in a given micro-ecological domain. We demonstrated that the competitive exclusion of a food limited bacteria is inevitable based on resource availability of species following the logistic growth. If the resource utilization capacity is equal then there is a possible coexistence and both bacteria co-operate with each other. This model can be used as a basic competition model for the prediction of the outcome of growth between any others microorganisms.

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1 INTRODUCTION

Comprehensive level of competition among various species is prevalent in every parity of natural world [1]. Human body harbors a diverse collection of microbial species, which undergoes fierce competition with one another for the establishment of successful colonization [2, 3, 4]. Numerous experiments with pure and mixed cultures have demonstrated various mechanisms by which bacteria can impair or kill other microorganisms [1, 3]. The deadly battle for survival in the resource limited habitat and the transfer of genes to the next generation serve as a driving force for diversification among the competing organisms [1]. Stunning level of microbial diversity has been observed by novel molecular techniques such as metagenomic sequencing and deep microbial tag sequencing [4]. Human oral cavity is one of the rich habitats for microbial colonization and it is estimated that > 700 species are present in this microecological niche [5, 6]. It is also regarded as the most dynamic microbial habitat in our body due to repeated cycles of food and water intake, frequent variation of temperature, pH, osmotic and oxygen tension [7].

Streptococcus mutans is the most dominating oral microbial species and is regarded as the primary causative agent of various dental diseases under certain conditions, although this bacterium exists as a normal microflora in dental biofilm community [6, 8, 9]. To establish a successful colonization in the oral cavity, this bacterium undergoes intense competition with other bacteria and this process is aided by its ability to produce acids from carbohydrate metabolism, to adhere and form biofilm on tooth surface, to tolerate the low pH and different environmental stresses in this fluctuating environment [7]. In addition, *S. mutans* can produce antimicrobial peptides to inhibit the growth of other competing species [10, 11]. It has been reported that *S. mutans* can inhibit the growth of other oral streptococci such as *S. pyogenes*, *S. gordonii*, *S. sanguis*, *S. oralis*, *Lactococcus lactis*, by producing antimicrobial peptides [10].

S. sanguis (sanguinis), which is also a member of the Gram positive, microaerophilic oral biofilm community as like *S. mutans* [12] and generally considered as a beneficial bacterium in oral habitat and coexist with other oral bacteria including *S. mutans* [13, 14]. However, *S. sanguis* can produce hydrogen peroxide to counter *S. mutans* [7]. This rivalry between *S. mutans* and *S. sanguis* at the microecological niche has been well-known for long times. Previous studies demonstrated that early colonization and increased numbers of *S. sanguis* in an infants oral cavity hinder the growth and colonization by *S. mutans* [14]. Likewise, high levels of *S. mutans* in the oral habitat correlate with reduced levels of *S. sanguis* [15]. Competitive exclusion between *S. mutans* and *S. sanguis* was also observed in germ-free rats model depending on inoculation pattern [16]. Thus coexistence and competition between *S. mutans* and *S. sanguis* depends on multiple factors [7]. Clinical studies have revealed that patients colonized by hydrogen peroxide producing *S. oligoformans* has reduced incidence of dental caries, caused by *S. mutans*, which instigated the need for investigation of the interaction between *S. mutans* and other oral streptococci [17].

Therefore, when a pathogenic species such as *S. mutans* is present in oral habitat with other non-harmful microorganisms, it would be difficult to predict the outcome of the interaction in terms of growth kinetics. A basic mathematical model for the growth of competing microbes would be important tool to describe the possible outcome [18]. In order to develop a basic model regarding the competition between two species, we need to consider two species for simplicity. Only after the establishment of a basic model for two competing species, further development of multispecies model would be possible. Several previous studies have reported the growth of a food-borne pathogen present in food using mathematical models [18, 19, 20]. A model for competition was developed between *Listeria monocytogenes* and *Lactococcus lactis* by considering the concentrations of lactic acid and pH [21]. Recently, a competition model

has been proposed based on the logistic model for *L. monocytogenes* and *L. lactis*, which could successfully describe the growth kinetics of these microorganisms [22]. However, the competition among n species has been reported for a regular reaction-diffusion model and it showed that the slow diffuser is the sole winner [23]. Instead of random diffusion, recently a new type of competitive and cooperative reaction-diffusion model has been developed [24, 25]. These models established some fundamental theories for any organisms with similar characteristics as long as the species diffuses according to per capita available resources.

A competition model was also introduced by considering the concept of critical population density for lactic acid bacteria against a competing bacterium [26]. However, these models were highly specific for the species of concern and can't be applied to other microbial species. Therefore, the goal of the present study was to develop a basic competition model based on *S. mutans* UA159 and *S. sanguis* ATCC 10556, which can be applied to other strains, for example *S. mutans* UA140 and *S. sanguinis* NY101. This model can also be applied for other species such as *S. pneumoniae*, *S. gordonii*, *S. pyogenes*, *S. mitis*, and *S. sobrinus* in mixed culture. This study offers three novel phenomena such as:

1. The new idea of modeling, where two species follow different growth laws and competing with each other.

2. By considering two bacterial populations, we observed the interactions between two organisms and established the results for coexistence of both species and extinction of one by the other.

3. The effect of crowding tolerance while the carrying capacities vary as well as the effects of intrinsic growth rates.

2 MATHEMATICAL MODEL AND METHODOLOGY

Taking into account of these microbial species as discussed in the previous section, we considered

two microbial species for logistic [27] and food-limited growth functions [28], using the following system of nonlinear differential equations

$$
\begin{cases}
\dfrac{du}{dt} &= \dfrac{r_1}{K_1} u(t) \left(K_1 - u(t) - w(t) \right) \\
\dfrac{dw}{dt} &= r_2 w(t) \left(\dfrac{K_2 - u(t) - w(t)}{K_2 + \beta(u(t) + w(t))} \right)
\end{cases} \quad (2.1)
$$

where, $u(t)$ is the concentration of microbial *S. mutans* UA159 and follows the logistic growth while $w(t)$ is the density of biomass of *S. sanguis* ATCC 10556 and satisfies the food limited growth function. For simplicity, throughout the paper we have omitted the strain numbers of bacteria, instead just mentioned the two species only. From biological literature, we defined all the parameters of (2.1) as follows:

$-K_1$ is the phase of the organism u and is growing at its maximum growth rate.
$-K_2$ is the phase of the organism w and is growing at its maximum growth rate.
$-r_1$ and r_2 are the specific growth rate for organisms u and w, respectively. In fact, the constants r_i, $(i = 1, 2)$, in this system are named the basic reproductive rate of population. It represents the instantaneous rate of change of population. However, after small modification of the model (2.1), the term containing the constant $\mu = r_i/K_i$ is the mortality. The parameter μ contributes negatively to the rate of change of the population and tends to make the population to be decreased.
$-\beta > 0$, a resource utilization factor and depending on the numerical value of β, it is determine which species is better competitor or whether coexistence is possible [29].

To expand the competition model, there are numerous examples of growth functions; as described by Gompertz and Gilpin-Ayala [30, 31].

2.1 Method of Analysis

For further mathematical analysis of the system (2.1), we considered the well known theorem established by Hartman-Grobman as described below:

Theorem 1. [32, 33] If the linearization matrix has no zero or pure imaginary eigenvalues then the phase portrait for the system near the

equilibria (u, v) can be obtained from the phase portrait of the linear system via a continuous change of coordinates.

Generally, it is a challenging task, even sometimes impossible to construct an analytic formula for the solution of (2.1). Instead, we consider the numerical solution or analyze the qualitative behavior of the solutions by considering the phase portrait (u, w). In many cases, the qualitative study is convenient to make the platform for constructing the numerical solutions. For further analysis, we considered the following solution methodologies:

1. Qualitative study of system (2.1), and
2. Numerical simulations in a series of examples.

For simplicity, we introduced the following two functions

$$f(u, w) = \frac{r_1}{K_1} u(t) \left(K_1 - u(t) - w(t) \right)$$

$$h(u, w) = r_2 w(t) \left(\frac{K_2 - u(t) - w(t)}{K_2 + \beta(u(t) + w(t))} \right)$$

and rewrite the problem (2.1) in the following path:

$$\begin{aligned} \frac{du}{dt} &= f(u, w) \\ \frac{dw}{dt} &= h(u, w). \end{aligned} \tag{2.2}$$

We also considered the possible solution of the system (2.2)

$$v(t) = \begin{pmatrix} u(t) \\ w(t) \end{pmatrix}$$

and it can be thought of as a parametrized curve and referred to as an integral curve of the problem. To show the outcome graphically in later sections the following factor is important.

$$f_u(u, w) = \frac{r_1(K_1 - 2u - w)}{K_1}$$

$$f_w(u, w) = -\frac{r_1 u}{K_1}$$

$$h_u(u, w) = -\frac{r_2 K_2 w(1 + \beta)}{(K_2 + \beta u + \beta w)^2}$$

$$h_w(u, w) = \frac{r_2(K_2^2 - K_2 u - 2K_2 w + \beta u K_2 - 2\beta u w - \beta u^2 - \beta w^2)}{(K_2 + \beta u + \beta w)^2}$$

- The vector form $(du/dt, dw/dt)$ is tangent to the solution path defined by $(u(t), w(t))$.
- The phase portraits that represent the family of solutions can be generated numerically or from the direction field.

It is remarked that the only limitations of either numerical approach or the direction filed are that the parameters in (2.2) need to be specified.

2.2 Equilibria and Linearization

As like first order non-linear system (2.2), we can obtain the solutions pattern near an steady state through linearization. Since (u^*, w^*) is an equilibrium, $f(u^*, w^*) = 0$ and simultaneously $h(u^*, w^*) = 0$ and the critical points are of (2.2) are

$$(u^*, w^*) = \{(0, 0), (K_1, 0), (0, K_2)\}.$$

It is seen that the coexistence steady state (u_s, w_s) of (2.2) is not straightforward; just hold on until numerical simulations as prescribed in Section 3.

At first, we rearrange the functions $f(u, w)$ and $h(u, w)$ in the following way,

$$\begin{aligned} f(u, w) &= \frac{r_1(uK_1 - u^2 - uw)}{K_1} \\ h(u, w) &= r_2 \left(\frac{K_2 w - uw - w^2}{K_2 + \beta u + \beta w} \right) \end{aligned} \tag{2.3}$$

and we obtained the linearized system

$$\begin{aligned} u' &= f_u(u^*, w^*)u(t) + f_w(u^*, w^*)w(t) \\ w' &= h_u(u^*, w^*)u(t) + h_w(u^*, w^*)w(t). \end{aligned}$$

To find the partial derivatives of $f(u, w)$ and $h(u, w)$, we employed

which yields the Jacobian matrix:

$$J_{(u,w)} = \begin{pmatrix} f_u & f_w \\ h_u & h_w \end{pmatrix}.$$

We also tested the stability analysis of (2.2) at the equilibira.

3 RESULTS AND DISCUSSION

Since we cannot solve the system (2.2) with demography analytically but it is possible to obtain some information about the behavior of the solutions. As the long-run behavior of solutions is important, following subsections deals with the matter of phase-plane analysis.

3.1 Qualitative Analysis

We considered phase portrait by considering the solution curves in the (u, w) plane.

1. Equilibrium $(0, 0)$**:** At the trivial equilibrium, the partial derivatives are

$$f_u(0,0) = r_1; \ f_w(0,0) = 0;$$

$$h_u(0,0) = 0 \text{ and } h_w(0,0) = r_2$$

which yields the linear system

$$\begin{aligned} \frac{du}{dt} &= r_1 u \\ \frac{dw}{dt} &= r_2 w \end{aligned} \tag{3.1}$$

and the corresponding Jacobian matrix at $(0, 0)$ is

$$J_{(0,0)} = \begin{pmatrix} r_1 & 0 \\ 0 & r_2 \end{pmatrix}.$$

The eigenvalues of $J_{(0,0)}$ are $\lambda_1 = r_1 > 0$, $\lambda_2 = r_2 > 0$. Put $r_1 = 10$ and $r_2 = 5$ which gives $\lambda_1 = 10, \lambda_2 = 5$ and the eigenvectors

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and } \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

respectively.

Since both eigenvalues of the matrix $J_{(0,0)}$ are real and positive which includes that the trivial steady state is always unstable. In the biological point of view, it means, when both of the bacterial organisms are present for competition in the same ecological habitat, they will repel and leave the sub-space of the niche until unless other factors are considered (Fig. 1).

2. Equilibrium $(0, K_2)$**:** After calculating the following terms

$$f_u(0, K_2) = \frac{r_1(K_1 - K_2)}{K_1}; \ f_w(0, K_2) = 0;$$

$$h_u(0, K_2) = -\frac{r_2}{(1+\beta)} \text{ and } h_w(0, K_2) = -\frac{r_2}{(1+\beta)}$$

the linearization matrix at $(0, K_2)$ is

$$J_{(0,K_2)} = \begin{pmatrix} \frac{r_1(K_1-K_2)}{K_1} & 0 \\ -\frac{r_2}{(1+\beta)} & -\frac{r_2}{(1+\beta)} \end{pmatrix}.$$

As seen, the eigenvalues of the matrix $J_{(0,K_2)}$ are $\lambda_1 = -r_2/(1+\beta) < 0$ and $\lambda_2 = r_1(K_1 - K_2)/K_1$, where λ_2 is strictly positive while $K_1 > K_2$ and strictly negative for $K_1 < K_2$. As an example, let us consider $\beta = 2$, $r_1 = 3$, $r_2 = 5$ and for $K_1 = 4, K_2 = 3$, we obtain $\lambda_1 = -5/3, \lambda_2 = 3/4$ and the corresponding eigenvectors

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \text{ and } \begin{pmatrix} -29/20 \\ 1 \end{pmatrix},$$

respectively. The two eigenvalues of the matrix $J_{(0,K_2)}$ are both real and have opposite signs. Thus the steady state is in the class of unstable saddle point as depicted in the left panel of Fig. 2. In oral microbial habitat, if *S. mutans* consumes higher level of resources, it will out-compete *S. sanguis* following the food limited growth. In this circumstance, *S. mutans* will utilize the resources and express genes required for bacteriocin production to inhibit the other bacteria and establish biofilm life style in the oral cavity. In a similar manner with identical β, r_1 and r_2 as above, let us assume $K_1 = 3$, $K_2 = 4$, and we have $\lambda_1 = -5/3, \lambda_2 = -1$ with respective eigenvectors

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \text{ and } \begin{pmatrix} -2/5 \\ 1 \end{pmatrix}.$$

Since both eigenvalues are negative, the equilibrium $(0, K_2)$ is an asymptotically stable node, see the right panel of Fig. 2. Thus, while *S. sanguis* has more accessibility to the natural resources, the bacteria *S. mutans* is in extinction.

3. Equilibrium $(K_1, 0)$**:** At the coordinates point

$(K_1, 0)$, the produced expressions translate to the linear system

$$f_u(K_1, 0) = -r_1; \; f_w(K_1, 0) = -r_1;$$

$$h_u(K_1, 0) = 0 \text{ and } h_w(K_1, 0) = \frac{r_2(K_2 - K_1)}{(K_2 + \beta K_1)}.$$

and the converted Jacobian matrix at $(K_1, 0)$ is

$$J_{(K_1,0)} = \begin{pmatrix} -r_1 & -r_1 \\ 0 & r_2(K_2 - K_1)/(K_2 + \beta K_1) \end{pmatrix}$$

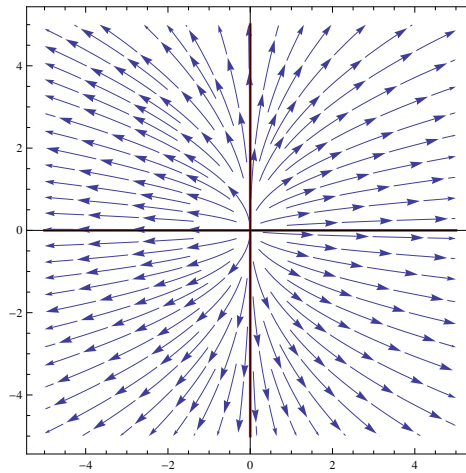$$\begin{pmatrix} u \\ w \end{pmatrix}' = \begin{pmatrix} c1 & n1 \\ n2 & c2 \end{pmatrix} \begin{pmatrix} u \\ w \end{pmatrix}$$

$$\begin{pmatrix} u \\ w \end{pmatrix}' = \begin{pmatrix} 10 & 0 \\ 0 & 5 \end{pmatrix} \begin{pmatrix} u \\ w \end{pmatrix}$$
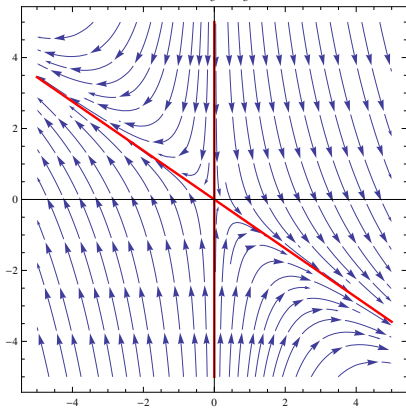

Fig. 1. The vector filed alongside the solutions of (3.1) near $(0, 0)$

$$\begin{pmatrix} u \\ w \end{pmatrix}' = \begin{pmatrix} c1 & n1 \\ n2 & c2 \end{pmatrix} \begin{pmatrix} u \\ w \end{pmatrix}$$

$$\begin{pmatrix} u \\ w \end{pmatrix}' = \begin{pmatrix} \frac{3}{4} & 0 \\ -\frac{5}{3} & -\frac{5}{3} \end{pmatrix} \begin{pmatrix} u \\ w \end{pmatrix}$$

$$\begin{pmatrix} u \\ w \end{pmatrix}' = \begin{pmatrix} c1 & n1 \\ n2 & c2 \end{pmatrix} \begin{pmatrix} u \\ w \end{pmatrix}$$

$$\begin{pmatrix} u \\ w \end{pmatrix}' = \begin{pmatrix} -1 & 0 \\ -\frac{5}{3} & -\frac{5}{3} \end{pmatrix} \begin{pmatrix} u \\ w \end{pmatrix}$$
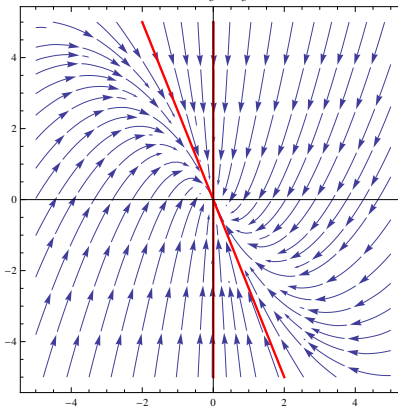

Fig. 2. The vector filed alongside the solutions of linear system for $\beta = 2$

The respective eigenvalues are $\lambda_1 = -r_1$ and $\lambda_2 = r_2(K_2 - K_1)/(K_2 + \beta K_1)$, where $\lambda_2 > 0$ if $K_2 > K_1$ and $\lambda_2 < 0$ for $K_2 < K_1$. Choosing the parameters $r_1 = 3$, $r_2 = 5$, $\beta = 3$, and for $K_1 = 3, K_2 = 4$, yields the eigenvalues $\lambda_1 = -3, \lambda_2 = 5/13$ with eigenvectors

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and } \begin{pmatrix} -39/44 \\ 1 \end{pmatrix},$$

respectively. The eigenvalues of the matrix $J_{(K_1,0)}$ are both real and have oppsite sign. Hence, according to theorem 1, we conclude that this critical point is an unstable saddle point as designed in the left diagram of Fig. 3. In terms of biology, *S. mutans* will be extinct at this competition scenario in this ecological niche. *S. sanguis* will have better resource access and it will activate its genetic machineries to produce hydrogen per oxide to kill *S. mutans*. Likewise, we have $\lambda_1 = -3, \lambda_2 = -5/13$ for $K_1 = 4, K_2 = 3$ with the equivalent parameters as defined above, and respective eigenvectors are

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and } \begin{pmatrix} -39/34 \\ 1 \end{pmatrix}.$$

At this point, both eigenvalues are negative, and we can summarize the equilibrium $(K_1, 0)$ is an asymptotically stable point (Fig. 3 (right)). So, in this circumstance, *S. sanguis* will be extinct from the given habitat.

Remark 1. One can consider the same study for $0 < \beta < 1$ to check the qualitative behavior of solutions and we left it to the reader. But wait until to check the numerical examples, where we consider this case for further investigation of the solutions.

Remark 2. For phase plane analysis, we used the software Wolfram MATHEMATICA 8 programming language.

3.2 Numerical Examples

In the following series of examples, different strategies were considered to select the parameters; sometimes carrying capacities are time-dependent and intrinsic growth rates as well. Here we observed few examples to check the solutions structure for various combinations of constant and periodic functions.

Example 1. At first, we considered $r_1 = r_2 = 2.0$, $\beta = 2.0$ and identical carrying capacity, $K_1 = K_2 = 3$. For equal initial densities $u_0 = w_0 = 0.4$, both *S. mutans* and *S. sanguis* are coexisting and the steady state solution (u_s, w_s) is shown in Fig. 4. Both bacteria are cooperating with each other when their resource distributions are equal even though the growth functions are different. Biologically, when nutrient become depleted, both of the strains will be affected and will be eradicated. But in the oral cavity, this scenario is unrealistic because we always take some food and drinks which replenish the nutrient scarcity and hence both species will persist.
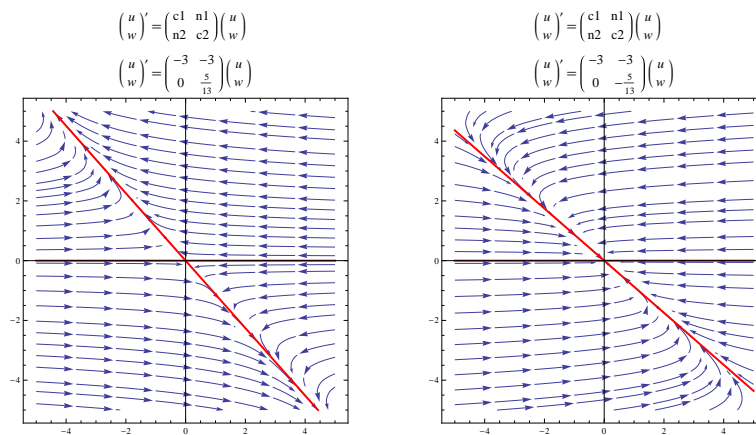

Fig. 3. The vector filed alongside the solutions of linear system for $\beta = 3$

Example 2. In this example, we considered $r_1 = r_2 = 2.0$ and two different periodic carrying capacities in left and right panels, see the caption in Fig. 5 for $\beta > 1$ and $0 < \beta < 1$. The results are plotted in Fig. 5 and the figure suggests that the species (*S. sanguis*) following the food-limited growth persists only. The main effect shows here for β and we conclude that the extinction rate of species w is faster for $\beta > 1$; for example $\beta = 2$. Moreover, the bacterial population exhibits oscillatory curves since the resource compositions and availability always fluctuates in the oral cavity, therefor it is realistic to show the solutions in a periodic pattern instead of smooth growth while the selected resource functions are fluctuating.

Example 3. At the current stage, we wanted to check the density level controlled by intrinsic growth rates by considering equal resource and identical local densities.

The Fig. 6 illustrates that the density of *S. mutans* is too low compare to the species *S. sanguis* due to the effect of $r_2 = 5$ and $0 < r_1 < r^* < r_2$ while $\beta = 0.5$. Biologically, the expectation is to get the higher population level for second microbial species since the growth rate is high.
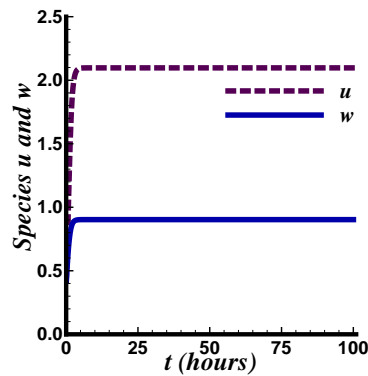

Fig. 4. Numerical solutions of (2.1) for $\beta = 2.0$, $r_1 = r_2 = 2.0$ **and** $K_1 = K_2 = 3.0$, **with initial values** $u_0 = w_0 = 0.4$, **where vertical directions represent the number of bacteria while time is in hours**
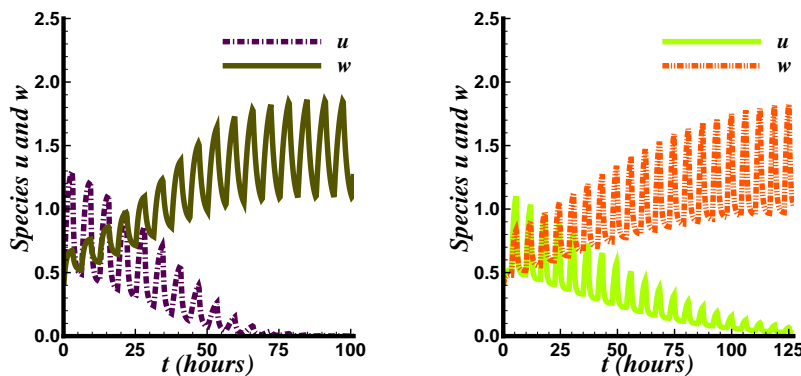

Fig. 5. Numerical solutions of (2.1) for $r_1 = r_2 = 2.0$ **and (left)** $\beta = 2.0$, $K_1 = K_2 = 2.0 + \sin(t)$, **(right)** $\beta = 0.5$, $K_1 = K_2 = \frac{3}{2+\sin(t)}$, **with** $u_0 = w_0 = 0.4$

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Example 4. Over the long term, it is interesting to observe the result for different carrying capacities as long as one is periodic rational and the rest is constant. For $r_1 = r_2 = 2.0$ and unequal carrying capacities, K_1, K_2, the solution of the logistic equation coincides with the carrying capacity K_1, while the solution of the Food-limited equation tends to 0 for $\beta = 0.5$ (see Fig. 7). In this example, we choose the carrying capacity K_1 for first species in a specific form and biologically it is interesting and meaningful in the sense that it varies with time. Since the resource is always limited in the environment, so we consider the function K_1 such that $0 < K_1 \leq c$, where c is a constant and approximately 6.6. It means

that the resource is always bounded, positive and variable with time. Our model suggests that even though one population is in extinction or minimum in number, the other population will fluctuate. It is justified in the perspective of oral cavity because this area is continuously expose to different nutrient intake, rapid flushes due to drinks, mechanical tooth brushing and anti-microbial agents used in toothpastes and mouth washes.

Example 5. Finally, by considering the function when both intrinsic growth rates and resource distributions are time-dependent for positive parameter $\beta < 1$. Two carrying capacities are
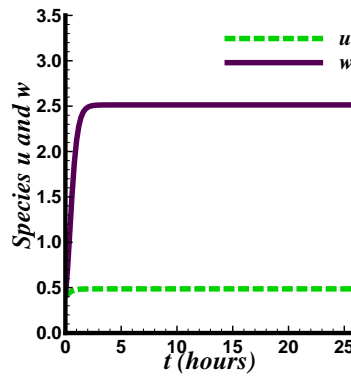

Fig. 6. Numerical solutions of (2.1) for $\beta = 0.5$, $r_1 = \frac{1}{2+\sin(t)}$, $r_2 = 5.0$ and $K_1 = K_2 = 3.0$, with $u_0 = w_0 = 0.4$
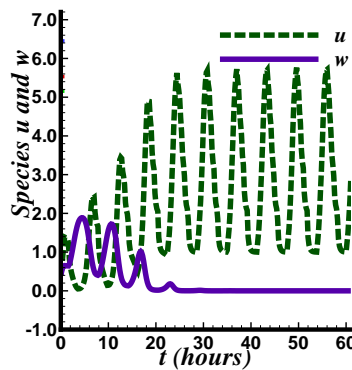

Fig. 7. Numerical illustrations of (2.1) for $\beta = 0.5$, $r_1 = r_2 = 2.0$ and $K_1 = \frac{3(2+\cos(t))}{2+\sin(t)}$, $K_2 = 2.0$, with initial values $u_0 = w_0 = 0.4$
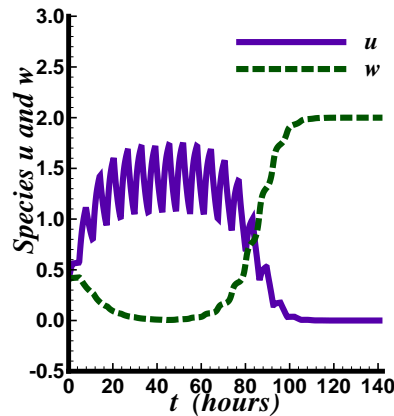
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Fig. 8. Numerical solutions (2.1) for $\beta = 0.5$**,** $r_1 = r_2 = \frac{1}{(2+\sin(t))}$ **and**
$K_1 = (2 + \cos(t))$, $K_2 = 2 - e^{-t}$**, with initial values** $u_0 = w_0 = 0.4$

$K_1 = (2 + \cos(t))$, $K_2 = 2 - e^{-t}$ and local population density $(u_0, w_0) = (0.4, 0.4)$ with equal growth rate. The results are depicted in the diagram 8 for densities of species u, w with respect to time. The plot focused that initially, the density of *S. mutans* is oscillating in the increasing rate and *S. sanguis* is eventually decreasing, density is too low and very close to zero. After certain time interval, the second species converges to the carrying capacity K_2 and the first one, u is in extinction. In this situation, *S. mutans* will face set back due to changed chemical environment or introduction of *S. mutans* specific bacteriophage which will cause the drastic reduction of its number whereas this changed environment will provide an opportunity for the growth of *S. sanguis*.

Remark 3. For numerical simulations, first we used the programming language FORTRAN (version plato 90/95) to produce the data and then to analyze the data, we consider TECPLOT 360 to draw the figures. It is also noted that we introduced the Runge-Kutta higher order numerical method to solve the non-linear system.

4   CONCLUSION

In this study, we described a new mathematical model to predict the possible outcome of a growth competing bacteria in a multi-species environment where they coexist or compete each other. We considered a system of non-linear equations with different growth laws. The considered model suggests that resource function plays an important role in determination of the competition outcome between two organisms in a given micro-ecological niche. If the carrying capacity is equal, there is a possible coexistence and both bacteria co-operate each other to sustain a multi-species biofilm life style. Moreover, it is remarkable that the parameter β and intrinsic growth rates are important factors to figure out the density level of populations. This model can also be applicable for other bacterial species in any habitats. Further microbiological experiments to investigate the effects of bacteriocin production capacity, biofilm formation ability, hydrogen peroxide production, resistance mechanism of the bacteria to these growth inhibitory agents on the competitive outcome should be performed to validate the proposed models. Finally, we have some open problems to the readers for further analysis:

1. To introduce different growth laws, for example Malthus and Gilpin-Ayala in (2.1) and study the problem;

2. To estimate the numerical values of β^* with $\beta \in (0, \beta^*]$ such that the species following the logistic growth law will persist only;

3. To add the diffusion term for both species in the problem (2.1) and analyze the revised model.

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COMPETING INTERESTS

Authors have declared that no competing interests exist.

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