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Implicit Four Step Stormer-cowell-type Method for General Second Order Ordinary Differential Equations

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Authors' contributions

This work was carried out in collaboration between the authors. Author SJK designed the study and wrote the first draft of the manuscript. Author OSI managed the analyses, numerical implementation of the study and the literature searches. All authors read and approved the final manuscript.

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ABSTRACT

An implicit four step stormer-cowell-typed method for direct solution of general second order ordinary differential equations is proposed in this study. In the derivation of the method, a combination of chebyshev and legendre polynomials was used as basis function to generate system of interpolation and collocation equations at selected grid points to obtain *systems* of equation. The resulting system of equations was solved for the unknown parameters and the values of these parameters were substituted into the approximate solution of the basis function. The required method was obtained by evaluating and simplify the resulting equations at the last end grid point of the step number. The resulting method is zero-stable, consistent and normalized. Numerical results compared favourably well with the existing methods.

Keywords: General second order; interpolation; collocation; order; zero stability; chebyshev; legendre.

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1. INTRODUCTION

This paper considers a method for the general second order ordinary differential equations of the form.

$$y'' = f(x, y, y'); \quad y(b) = \alpha ; y'(b) = \beta , \quad f \in c^2(a, b) \tag{1}$$

where f is a given real-valued function which is continuous within the interval of integration.

It has been noted that an analytical solution to (1) is of little value because many of such problem cannot be solved by an analytical approach. In practice, the problem (1) is reduced to systems of first-order equations and any methods for first order equation are used to solve them. Awoyemi [4], Kayode [7], extensively discussed bulky dimension of the problem after it has been reduced to a system of first-order equations, which wasted a lot of human effort and computer time. Implicit linear multistep methods which have better stability condition is generally adopted in the predictor-corrector method. The major setback of the method is that the predictors are in dropping the order of accuracy hence it has effects on the accuracy of the method. Later, block scheme was adopted to cater for the setback of predictor-corrector methods. This method was revealed to have the possessions of Runge-Kutta for being self-started and also gives an autonomous solution without overlapping. The challenge of block method is that it involves computational burdens with the use of more time for computation. Scholars who have proposed methods from predictor-corrector method and block method among them are Kayode and Obarhua [10], Kayode and Adeyeye [9], Kayode [3], Kayode and Obarhua [12], Adesanya et al. [1], Yahaya and Badmus [11], Awoyemi et al. [5], Badmus and Yahaya [6] to mention but few. In this paper, we proposed an implicit four step stormer- cowell type method which is a linear multistep method. The combination of Chebyshev and legendry polynomial was used as basis function in generating the interpolation and collocation equations for the development of the method for the solution of (1).

2. DERIVATION OF THE METHOD

The derived implicit four step stormer-cowell type method with continuous coefficients for the solution of (1) is of the form

$$\sum_{j=k-2}^{k} \alpha_j y_{n+j}(x) = h^2 \sum_{j=0}^{k} \beta_j f_{n+j} \tag{2}$$

where α_j, β_j are continuous functions of

$$x, \; y_{n+j}, \approx y(x_{n+j}), f_{n+j} = f(x_{n+j}, y_{n+j}, y_{n+j}),$$

k is the step number of the method and h is the step size of the method.

In the work, we considered the combination of Chebyshev and Legendre Polynomials in the form

$$y(x) = \sum_{j=0}^{c+i-1} a_j \left[T_j(x) + P_j(x) \right] \tag{3}$$

where $T_j(x)$ is the Chebyshev polynomial of the first kind and $P_j(x)$ is the Legendre polynomial. Equation (3) is the basis function with a single variable x, where $x \in (a,b)$, a' s are real unknown parameter to be determined and $c + i$ is the sum of collocation and interpolation points, c is the number of collocation points and i is the number of interpolation points.

The second derivative of (3) is

$$y''(x) = \sum_{j=0}^{c+i-1} a_j \left[T_j''(x) + P_j''(x) \right] \tag{4}$$

Using equation (4) in (1) to have

$$y''(x) = \sum_{j=0}^{c+i-1} a_j T_j''(x) + \sum_{j=0}^{c+i-1} a_j P_j''(x) = f(x, y, y'), \tag{5}$$

Equations (3) and (4) will be respectively interpolated and collocated to obtain the required method for step number k.

Collocated (4) at $x_{n+j}, j = 0(1)4$ and interpolated (3) at $x_{n+j}, j = 2,3$ resulted in the following set of equations

$$f_n = \left(\begin{array}{l} 7a_2 + \frac{78}{2}a_3x_n + \frac{1188}{8}a_4x_n^{2} - \frac{188}{4}a_4 + \frac{3820}{8}a_5x_n^{3} \\ -\frac{1380}{8}a_5x_n + \frac{22290}{16}a_6x_n^{4} - \frac{12996}{16}a_6x_n^{2} + \frac{786}{16}a_6 \end{array} \right)$$

$$f_{n+1} = \left(\begin{array}{l} 7a_2 + \frac{78}{2}a_3x_{n+1} + \frac{1188}{8}a_4x^{2}{}_{n+1} - \frac{188}{4}a_4 + \frac{3820}{8}a_5x^{3}{}_{n+1} \\ -\frac{1380}{8}a_5x_{n+1} + \frac{22290}{16}a_6x^{4}{}_{n+1} - \frac{12996}{16}a_6x^{2}{}_{n+1} + \frac{786}{16}a_6 \end{array} \right)$$

$$f_{n+2} = \left(\begin{array}{l} 7a_2 + \frac{78}{2}a_3x_{n+2} + \frac{1188}{8}a_4x^{2}{}_{n+2} - \frac{188}{4}a_4 + \frac{3820}{8}a_5x^{3}{}_{n+2} \\ -\frac{1380}{8}a_5x_{n+2} + \frac{22290}{16}a_6x^{4}{}_{n+2} - \frac{12996}{16}a_6x^{2}{}_{n+2} + \frac{786}{16}a_6 \end{array} \right)$$

$$f_{n+3} = \left(\begin{array}{l} 7a_2 + \frac{78}{2}a_3x_{n+3} + \frac{1188}{8}a_4x^{3}{}_{n+3} - \frac{188}{4}a_4 + \frac{3820}{8}a_5x^{3}{}_{n+3} \\ -\frac{1380}{8}a_5x_{n+3} + \frac{22290}{16}a_6x^{4}{}_{n+3} - \frac{12996}{16}a_6x^{2}{}_{n+3} + \frac{786}{16}a_6 \end{array} \right) \qquad (6)$$

$$f_{n+4} = \left(\begin{array}{l} 7a_2 + \frac{78}{2}a_3x_{n+4} + \frac{1188}{8}a_4x^{2}{}_{n+4} - \frac{188}{4}a_4 + \frac{3820}{8}a_5x^{3}{}_{n+4} \\ -\frac{1380}{8}a_5x_{n+4} + \frac{22290}{16}a_6x^{4}{}_{n+4} - \frac{12996}{16}a_6x^{2}{}_{n+4} + \frac{786}{16}a_6 \end{array} \right)$$

$$y_{n+3} = \left(\begin{array}{l} 2a_0 + 2a_1x_{n+3} + \frac{7}{2}a_2x^{2}{}_{n+3} - \frac{3}{2}a_2 + \frac{13}{2}a_3x^{3}{}_{n+3} - \frac{9}{2}a_3x_{n+3} + \\ \frac{99}{8}a_4x^{4}{}_{n+3} - \frac{94}{8}a_4x^{2}{}_{n+3} + \frac{11}{8}a_4 + \frac{191}{8}a_5x^{5}{}_{n+3} - \frac{230}{8}a_5x^{3}{}_{n+3} + \frac{55}{8}a_5x_{n+3} \\ + \frac{743}{16}a_6x^{6}{}_{n+3} - \frac{1083}{16}a_6x^{4}{}_{n+3} + \frac{393}{16}a_6x^{2}{}_{n+3} - \frac{21}{16}a_6 \end{array} \right)$$

$$y_{n+2} = \left(\begin{array}{l} 2a_0 + 2a_1x_{n+2} + \frac{7}{2}a_2x^{2}{}_{n+2} - \frac{3}{2}a_2 + \frac{13}{2}a_3x^{3}{}_{n+2} - \frac{9}{2}a_3x_{n+2} + \\ \frac{99}{8}a_4x^{4}{}_{n+2} - \frac{94}{8}a_4x^{2}{}_{n+2} + \frac{11}{8}a_4 + \frac{191}{8}a_5x^{5}{}_{n+2} - \frac{230}{8}a_5x^{3}{}_{n+2} + \frac{55}{8}a_5x_{n+2} \\ + \frac{743}{16}a_6x^{6}{}_{n+2} - \frac{1083}{16}a_6x^{4}{}_{n+2} + \frac{393}{16}a_6x^{2}{}_{n+2} - \frac{21}{16}a_6 \end{array} \right)$$

The system of equation (6) is solved by Gaussian elimination method to obtain the value of the unknown parameters a_j ,(j= 0,1,2,3,4,5,6) as follows

$$a_0 = \frac{1}{2}\left(\begin{array}{l} y_{n+2} - 2a_1x_{n+2} - \frac{7}{2}a_2x^{2}{}_{n+2} + \frac{3}{2}a_2 - \frac{13}{2}a_3x^{3}{}_{n+2} + \frac{9}{2}a_3x_{n+2} - \\ \frac{99}{8}a_4x^{4}{}_{n+2} + \frac{94}{8}a_4x^{2}{}_{n+2} - \frac{11}{8}a_4 - \frac{191}{8}a_5x^{5}{}_{n+2} + \frac{230}{8}a_5x^{3}{}_{n+2} - \frac{55}{8}a_5x_{n+2} \\ -\frac{743}{16}a_6x^{6}{}_{n+2} + \frac{1083}{16}a_6x^{4}{}_{n+2} - \frac{393}{16}a_6x^{2}{}_{n+2} + \frac{21}{16}a_6 \end{array} \right)$$

$$a_1 = \frac{1}{2h}\begin{pmatrix} y_{n+3} - y_{n+2} - \frac{14}{2}a_2x_nh - \frac{35}{2}a_2h^2 - \frac{393}{2}a_3x_n^{\ 2}h - \frac{195}{2}a_3x_nh^2 - \frac{247}{2}a_3h^3 + \frac{9}{2}x_3h \\ -\frac{396}{8}a_4x_n^{\ 3`}h - \frac{2970}{8}a_4x_n^{\ 2}h^2 + \frac{7524}{8}a_4x_nh^3 - \frac{6435}{8}a_4h^4 + \frac{188}{8}a_4x_nh - \frac{470}{8}a_4h^2 \\ -\frac{955}{8}a_5x_n^{\ 4}h - \frac{9550}{8}a_5x_n^{\ 3}h^2 - \frac{3629}{8}a_5x_n^{\ 2}h^3 - \frac{62075}{8}a_5x_nh^4 - \frac{40301}{8}a_5h^5 \\ +\frac{690}{8}a_5x_n^{\ 2}h + \frac{3450}{8}a_5x_nh^2 + \frac{4370}{8}a_5h^3 - \frac{55}{8}a_5h + \frac{46458}{16}a_6x_n^{\ 5}h \\ +\frac{55725}{16}a_6x_n^{\ 4}h^2 + \frac{14860}{16}a_6x^3{}_nh^3 - \frac{724425}{16}a_6x^2{}_nh^4 - \frac{940638}{16}a_6x_nh^5 \\ +\frac{494095}{16}a_6h^5 + \frac{4332}{16}a_6x^3{}_nh^6 + \frac{32490}{16}a_6x^2{}_nh^2 - \frac{70395}{16}a_6h - \\ \frac{786}{16}a_6x_nh - \frac{1965}{16}a_6h^2 \end{pmatrix}$$

$$a_2 = \frac{1}{7h}\begin{pmatrix} f_n - \frac{78}{2}a_3x_n - \frac{1188}{2}x_n^{\ 2}a_4 + \frac{188}{8}a_4 - \frac{3820}{8}x_n^{\ 3}a_4 + \frac{1380}{8}x_n\ a_5 - \frac{22290}{16}a_5x_n^{\ 4} \\ -\frac{12996}{16}a_6x_n^{\ 2} + \frac{786}{16}a_6 \end{pmatrix} \tag{7}$$

$$a_3 = \frac{2}{78h}\begin{pmatrix} f_{n+1} - f_n - \frac{2376}{8}a_4x_nh - \frac{1188}{8}a_4h^2 - \frac{11460}{8}a_5x_n^{\ 2}h - \frac{11460}{8}a_5x_nh^2 \\ -\frac{3820}{8}a_5h^3 + \frac{1380}{8}a_5h - \frac{89160}{16}a_6x_n^{\ 3}h - \frac{133740}{16}a_6x_n^{\ 2}h^2 - \frac{89160}{16}a_6x_nh^3 \\ -\frac{22290}{16}a_6h^4 + \frac{25992}{16}a_6x_nh + \frac{12996}{8}a_6h^2 \end{pmatrix}$$

$$a_4 = \frac{8}{2376h^2}\begin{pmatrix} f_{n+2} - 2f_{n+1} + f_n - \frac{22920}{8}a_5x_nh^2 - \frac{22920}{8}a_5h^3 - \frac{267480}{16}x^2{}_nh^2a_6 \\ -\frac{534960}{16}a_6x_nh^3 - \frac{312060}{16}a_6h^4 \end{pmatrix}$$

$$a_5 = \frac{8}{22920h^2}\left(f_{n+3} - 3f_{n+2} + 3f_{n+1} - f_n - \frac{534960}{16}x_nh^3a_6 - \frac{802440}{16}h^4a_6\right)$$

$$a_6 = \frac{16}{534960h^2}\left(f_{n+4} - 4f_{n+3} + 4f_{n+2} - 4f_{n+1} + f_n\right)$$

They a_j's are substituted back into (3) and simplifying to give a continuous method of the type

$$y_{n+k}(x) = \sum_{j=k-2}^{k-1}\alpha_j y_{n+j}(x) + h^2\sum_{j=0}^{k}\beta_j(x)f_{n+j} \tag{8}$$

Applying the transformation in Kayode & Obarhua (2015), $t = \frac{x - x_{n+k-1}}{h}$ and $dt = \frac{1}{h}dh$, (8)

becomes

$$y_{n+4} - (t+1)y_{n+3} + ty_{n+2} = h^2 \left(\begin{array}{l} \left(\begin{array}{l} -\frac{980}{360} - \frac{210}{360}t + \frac{160}{360}t^2 + \frac{150}{360}t^3 - \frac{160}{360}t^4 \\ +\frac{280}{360}t^5 - \frac{20}{360}t^6 \end{array} \right) f_n \\ + \left(\begin{array}{l} \frac{2806}{5760} + \frac{186}{24}t + \frac{218}{120}t^2 - \frac{125}{240}t^3 - \frac{160}{960}t^4 \\ -\frac{420}{240}t^5 + \frac{180}{24}t^6 \end{array} \right) f_{n+1} \\ + \left(\begin{array}{l} -\frac{205}{11520} + \frac{243}{960}t - \frac{483}{24}t^2 + \frac{353}{96}t^3 + \frac{182}{120}t^4 \\ +\frac{14}{196}t^5 + \frac{182}{96}t^6 \end{array} \right) f_{n+2} \\ + \left(\begin{array}{l} \frac{1206}{5760} - \frac{283}{12}t + \frac{353}{120}t^2 + \frac{288}{960}t^3 + \frac{330}{120}t^4 \\ -\frac{320}{24}t^5 + \frac{88}{240}t^6 \end{array} \right) f_{n+3} \\ + \left(\begin{array}{l} \frac{245}{5760} - \frac{183}{240}t - \frac{108}{12}t^2 + \frac{358}{240}t^3 - \frac{280}{960}t^4 \\ +\frac{66}{120}t^5 + \frac{18}{2880}t^6 \end{array} \right) f_{n+4} \end{array} \right) \tag{9}$$

The coefficients are given as follows

$$\alpha_2 = t$$

$$\alpha_3 = -(t+1)$$

$$\beta_0 = h^2 \left(\begin{array}{l} -\frac{980}{360} - \frac{210}{360}t + \frac{160}{360}t^2 + \frac{150}{360}t^3 - \frac{160}{360}t^4 \\ +\frac{280}{360}t^5 - \frac{20}{360}t^6 \end{array} \right)$$

$$\beta_1 = h^2 \left(\begin{array}{l} \frac{2806}{5760} + \frac{186}{24}t + \frac{218}{120}t^2 - \frac{125}{240}t^3 - \frac{160}{960}t^4 \\ -\frac{420}{240}t^5 + \frac{180}{24}t^6 \end{array} \right)$$

$$\beta_2' = h \left(\begin{array}{l} \frac{243}{960}t - \frac{483}{24}t^2 + \frac{353}{96}t^3 + \frac{182}{120}t^4 \\ +\frac{14}{196}t^5 + \frac{182}{96}t^6 \end{array} \right)$$

(10)

$$\beta_3 = h^2 \left(\begin{array}{l} \frac{1206}{5760} - \frac{283}{12}t + \frac{353}{120}t^2 + \frac{288}{960}t^3 + \frac{330}{120}t^4 \\ -\frac{320}{24}t^5 + \frac{88}{240}t^6 \end{array} \right)$$

$$\beta_4 = h^2 \left(\begin{array}{l} \frac{245}{5760} - \frac{183}{240}t - \frac{108}{12}t^2 + \frac{358}{240}t^3 - \frac{280}{960}t^4 \\ +\frac{66}{120}t^5 + \frac{18}{2880}t^6 \end{array} \right)$$

The first derivative of (10) are as follows

$$\alpha_2^{'} = \frac{1}{h}$$

$$\alpha_3^{'} = -\frac{1}{h}$$

$$\beta_0^{'} = h\begin{pmatrix} \frac{210}{360} + \frac{320}{360}t + \frac{450}{360}t^2 - \frac{640}{360}t^3 \\ + \frac{14400}{360}t^4 - \frac{120}{360}t^5 \end{pmatrix}$$

$$\beta_1^{'} = h\begin{pmatrix} \frac{186}{24} + \frac{436}{120}t + \frac{375}{240}t^2 - \frac{640}{960}t^3 \\ - \frac{2100}{240}t^4 + \frac{1080}{24}t^5 \end{pmatrix}$$

$$\beta_2^{'} = h\begin{pmatrix} \frac{243}{960} - \frac{966}{24}t + \frac{1059}{96}t^2 + \frac{728}{120}t^3 \\ + \frac{70}{196}t^4 + \frac{1092}{96}t^5 \end{pmatrix} \tag{11}$$

$$\beta_3^{'} = h\begin{pmatrix} -\frac{283}{12} + \frac{706}{120}t + \frac{864}{960}t^2 + \frac{1320}{120}t^3 \\ - \frac{1600}{24}t^4 + \frac{528}{240}t^5 \end{pmatrix}$$

$$\beta_4^{'} = h\begin{pmatrix} -\frac{183}{240} + \frac{216}{12}t + \frac{1074}{240}t^2 - \frac{1120}{960}t^3 \\ + \frac{330}{120}t^4 + \frac{158}{2880}t^5 \end{pmatrix}$$

Evaluating (10) and (11) at $t = 1$ which implies that $x = x_{n+4}$ gives discrete scheme

$$y_{n+4} = 2y_{n+3} - y_{n+2} + \frac{h^2}{240}\left(19f_{n+4} + 204f_{n+3} + 14f_{n+2} + 4f_{n+1} - f_n\right) \tag{12}$$

The first derivative

$$y'_{n+4} = \frac{1}{h}\left(y_{n+3} - y_{n+2}\right) + \frac{h}{1440}\left(481f_{n+4} + 1764f_{n+3} - 198f_{n+2} + 140f_{n+1} - 27f_n\right) \tag{13}$$

The Predictor

$$y_{n+4} = 2y_{n+3} - y_{n+2} + \frac{h^2}{240}\left(140f_{n+3} + 197f_{n+2} - 156f_{n+1} + 59f_n\right) \tag{14}$$

with its first derivative as

$$y'_{n+4} = \frac{1}{h}\left(-y_{n+2}+y_{n+3}\right)+\frac{h}{360}\left(922f_{n+3}-771f_{n+2}+516f_{n+1}-127f_n\right) \tag{15}$$

Other explicit schemes were generated to evaluate the remaining values using Taylor series.

$$y_{n+j} = y_n + (jh)y_n' + \frac{(jh)^2}{2!}f_n + \frac{(jh)^3}{3!}\left\{\frac{\partial f_n}{\partial x_n} + y'_n\frac{\partial f_n}{\partial y_n} + f_n\frac{\partial f_n}{\partial y'_n}\right\} + 0\left(h^4\right)$$

And

$$y'_{n+j} = y'_n + (jh)f_n + \frac{(jh)^2}{2!}\left\{\frac{\partial f_n}{\partial x_n} + y'_n\frac{\partial f_n}{\partial y_n} + f_n\frac{\partial f_n}{\partial y'_n}\right\} + 0\left(h^3\right)$$

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3. ANALYSIS OF BASIC PROPERTIES

3.1 Order and Error Constant of the Method

Let the linear differences operator L associated with the continuous multistep method 2 be defined as:

where $y(x)$ is an arbitrary function, continuously

$$L[y(x)\;;\;h] = \sum_{j=0}^{k}\left[\alpha_j y(x+jh) - h^2\beta_j y''(x+jh)\right]$$

differentiable in the interval (a, b)

Expanding $y(x_n+jh)$ and $y''(x_n+jh)$ as Taylor series about x and collecting terms

$$L[y(x);h] = C_0 y(x) + C_1 h y^1(x) + C_2 h^2 y^{(2)} + \ldots + C_P h^p y^{(p)}(x)$$

where the C_p are constants, Lambert [13]. The difference operator L and the associated continuous linear multistep method (2) are said to be of order P if in (2)

$$C_0 = C_1 = C_2 = \ldots = C_{P+1} = 0 \;\; and \;\; C_{P+2} \neq 0$$

The term C_{P+2} is called the error constant and it implies that the local truncation error is given by $C_{P+2}h^{p+2}y^{(p+2)}(x_n)$

For our method

$$C_0 = C_1 = C_2 = C_3 = C_4 = 0, C_5 = C_6 = 0, C_7 = -\frac{1}{240},$$

Therefore the derived scheme is of order 5

4. THE CONSISTENCY OF THE METHOD

Definition: The method is said to be consistent if it has an order of at least one. If we define the first and second characteristic polynomial

$$\rho(x) = \sum_{j=0}^{k} \alpha_j z^j \tag{16}$$

$$\sigma(x) = \sum_{j=0}^{k} \alpha_j z^j \tag{17}$$

where z is the principal root, $\alpha_j \neq 0 \;\; and \;\; \alpha_0^{\,2} + \beta_0^{\,2} \neq 0$

Definition: The linear multistep method (2) is said to be consistent, if it satisfies the following condition, Lambert [13]

(i) the order $\rho \geq 1$

(ii) $\sum_{j=0}^{k} \alpha_j = 0$

(iii) $\rho(1) = \rho'(1) = 0$

(iv) $\rho''(1) = 2\;!\sigma(1)$

For our method

Condition (i) is satisfied since the scheme is of order 5

Condition (ii) is satisfied since

$$\alpha_0 + \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 = 0;\; 0+0+1-2+1=0$$

Condition (iii) is satisfied since

$$\rho(r) = r^4 - 2r^3 + r^2 \, and \, \rho'(r) = 4r^3 - 6r^2 + 2r$$

where r =1; $\rho(r) = \rho'(r) = 0$

Condition (iv) is satisfied since

$$\rho''(r) = 12r^2 - 12r + 2 \; and \, \sigma(r) = \frac{1}{240}\left(19r^4 + 204r^3 + 14r^2 + 4r - 1\right)$$

where r =1;

$$\sigma(1) = 2! \times \left(\frac{19}{240} + \frac{204}{240} + \frac{14}{240} + \frac{4}{240} - \frac{1}{240}\right) = 2 \times \frac{240}{240} = 2 \times 1 = 2$$

Therefore $\rho''(r) = 2 \; ! \sigma(r) = 2$

Hence the four condition are satisfied, the method is consistent

5. ZERO STABILITY

Definition: A linear multistep method is said to be zero-stable, if no root of the first characteristics polynomial $\rho(r)$ has a modulus greater than one and if every root of modulus one has multiplicity not greater than two. The scheme is zero stable when no root of the first characteristics polynomial has a modulus greater than one that is.

A method is zero stable if $\rho(x) = \sum_{j=0}^{k} \alpha_j = 0$,

where α_j are the coefficients of $\sum_{j=0}^{k} \alpha_j y_{n+j}$

$$\sum_{j=0}^{k} \alpha_j = \alpha_0 + \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 = 0+0+0+1-2+1 = 0$$

$$\rho(r) = r^4 - 2r^3 + r^2 = 0$$
$$(r^2 - 2r + 1) = 0$$
$$(r - r)(r - 1) = 0$$
$$r = 1 \;\; twice$$

Thus, the method is zero stable.

6. STABILITY INTERNAL

The Equation (11) is said to be stable if for a given h all the roots z_n of the characteristics Polynomial $\sigma(z, \hbar) = \rho(z) + h\sigma(z) = 0$ satisfies $|z_s| \prec 1, \; s = 1, 2 \ldots\ldots\ldots, n \; where \;\; \hbar = \lambda h$

We take on the boundary locus method to determine the stability interval. Substituting the test equation $y' = -\lambda y$ into Equation (11) provides $\hbar(r, h) = \frac{\rho(r)}{\sigma(r)}, \quad r = e^{i\theta}$. After simplification, the stability interval gives $(-4.528 \; , \; 0)$ after evaluating $\hbar(r, h)$ the interval $(0^0, \; 180^0)$.

7. IMPLEMENTATION OF THE METHOD

Problem 1

We consider a linear second order ordinary differential equation

$$y'' = y' \; y(0) = 0, \quad y'(0) = -1, \quad h = 0.1$$

$$Exact \; solution : y(x) = 1 - \exp(x)$$

Problem 2

We consider a linear second order ordinary differential equation

$$y'' = 100y$$
$$y(0) = 1, \quad y'(0) = -10, \quad h = 0.01$$
$$Exact \; solution : y(x) = e^{-10x}$$

Problem 3

We consider a non-linear second order ordinary differential equation

$$y'' - x(y')^2 = 0$$
$$y(0) = 1, \quad y'(0) = \frac{1}{2}, \quad h = 0.003125$$
$$Exact \; solution : y(x) = 1 + \frac{1}{2} In\left(\frac{2+x}{2-x}\right)$$

7.1 Shown in the Tables 1-3 are Numerical Results to Problems 1-3

The computational error of our method tested on problems 1-3 compared to other researchers. Problem 1 was compared with

Table 1. Table for problem 1

X	Error in Kayode and Adeyeye [9]	Error in new Method
0.4	8.17176E-07	8.025651E-9
0.5	3.10356E-06	1.662684E-8
0.6	6.56957E-06	2.073986E-8
0.7	1.14380E-05	3.421021E-8
0.8	1.79656E-05	4.332165E-8
0.9	2.64474E-05	6.285674E-8
1.0	3.72222E-05	7.628358E-8
1.1	5.06786E-05	1.053582E-7
1.2	6.72615E-05	1.273987E-7

Table 2.Table for problem 2

X	Error in Awari [2]	Error in new Method
0.1	1.353E-7	3.674E-10
0.2	3.658E-7	1.287E-10
0.3	6.051E-7	2.364E-9
0.4	8.502E-7	2.957E-9
0.5	1.104E-7	2.900E-9
0.6	1.369E-6	3.405E-9
0.7	1.450E-6	4.176E-9
0.8	1.597E-6	4.493E-9
0.9	1.763E-6	4.416E-9
1.0	1.946E-6	4.756E-9
1.1	2.099E-6	5.165E-9
1.2	2.374E-6	5.454E-9

Table 3. Table for problem 3

X	Error in Kayode and Adeyeye [8]	Error in Awoyemi et al. [5]	Error in new method
0.1	4.831380E-11	1.0264E-11	0.000000E+00
0.2	3.382836E-09	7.9716E-11	1.638689E-13
0.3	1.580320E-08	2.6111E-10	3.179679E-13
0.4	4.333951E-08	6.5442E-10	8.195666E-13
0.5	9.391426E-08	1.3893E-09	1.321387E-12

Kayode and Adeyeye [9]. Problem 2 was compared with Awari [2], while the result of Problem 3 was compared with Kayode and Adeyeye [8] and Awoyemi et al. [5].

4. CONCLUSIONS

In this paper, we have derived, investigated and implemented an implicit four step stormer-cowell type method for the solution of general second order ordinary differential equations by adopting a combination of Chebyshev and Legendre polynomials as the basis function. Collocation and interpolation approach is adopted for the derivation of the method. It has also been shown that the method is consistent, zero stable hence convergent. In Table 1, our method performs better than Kayode and Adeyeye [9], Table 2; showed improved accuracy than Awari [2] , also Table 3; showed that our method perform better than Kayode and Adeyeye [8], Awoyemi et al [5]. Hence, our method compared favorably with the existing methods.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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