



Effect of Channel Slope on Hydraulic Jump Characteristics

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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ABSTRACT

Hydraulic jump mainly serves as an energy dissipater to dissipate excess energy of flowing water downstream of hydraulic structures, such as spillway, sluice gates etc., This research paper investigates the effect of channel slope on the characteristics of free hydraulic jump and the energy dissipation downstream the gate. Generally, this research investigates the main characteristics and parameters of the free hydraulic jump such as; the sequent depth, the relative hydraulic jump length and the relative distance to the jump. In the present research paper, these characteristics were tested in rectangular channel downstream (DS) the vertical gate. The experimental program was conducted on a re-circulating flume with 2.5 m long, 9 cm wide and 30 cm deep; with discharges range from 3 to 230 LPM. Statistical equation was developed to correlate the length, sequent depth ratio and distance of jump with the other independent parameters. Finally, clear matching of results from the length of jump was obtained.

Keywords: Open channel; hydraulic jump; energy dissipation; sloping channel; sluice gate; water structure.

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List of Symbols

Fr_1	Inflow Froude number [-]
Q	Flow discharge [$m^3.s^{-1}$]
g	Acceleration due to gravity [$m.s^{-2}$]
Y_1	Upstream sequent depth [m]
Y_2	Downstream sequent depth [m]
S_o	Channel slope [-]
L_{Hj}	Length of jump [m]
Dis	Distance to jump [m]

1. INTRODUCTION

Downstream of many types of hydraulic structures such as dams and barrages, sluice gates and draft tubes of hydraulic turbines, a considerable portion of the kinetic energy in supercritical flow must be dissipated to prevent scour and erosion. Chern [1] Study the effect of a corrugated bed on the hydraulic jump, a smoothed particle hydrodynamics (SPH) model is applied to investigate the characteristics of hydraulic jumps in various corrugated beds. It is found that the sinusoidal bed can dissipate more energy than other beds. As a result, corrugated beds can be used to enhance energy dissipation of hydraulic jump in the open channel. In general, the proposed SPH model is capable of simulating the effect of corrugated beds on hydraulic jump characteristics. Kordi [2] study the transitional expanding hydraulic jump. The results indicate that the post depth Y_2 required to form an expanding jump is distinctly smaller than that for the corresponding classical jumps. The expanding jump length was 1.25 times the corresponding free jump length. A jump formed in a horizontal, wide rectangular channel with a smooth bed is often referred to as the classical hydraulic jump and has been studied extensively [3,4,5,6]. Hager [7] analyzed the hydraulic jump theoretically and experimental means.

Hughes [8] studied the hydraulic jump characteristics over several artificially roughened test beds in a horizontal rectangular flume with smooth side walls. Observations showed that boundary roughness reduces both the sequent depth and the length of a hydraulic jump, and that the observed reductions were related to both Froude number and the degree of roughness. The observed hydraulic jump characteristics were consistent with theory, and a proposed approximation for a theoretical hydraulic jump equation was found to compare favorably with the observed characteristics. Afzal [9] investigated the stream-wise flow structure of a turbulent hydraulic jump over a rough bed

rectangular channel. The hydraulic jump over a rough bed channel can be directly deduced from classical smooth bed hydraulic jump theory, provided the upstream Froude number is replaced by the effective upstream Froude number. Abdel-Azim [10] study the effect of both positive and negative slopes on the hydraulic jump. The analysis of results indicated that both the inlet Froude number and the bottom slope have major effects while the inflow depth ratio has a minor effect on the depth ratio of the jump at the outlet. Chyan-Deng [11], and Smith [12] Study the hydraulic jump in an inclined rectangular chute contraction. They developed theoretical equations for the sequent-area and sequent-depth ratios for hydraulic jumps in the contraction considering the effects of contracting width and sloping bottom. Beirami [13] studied the hydraulic jumps in sloping channels and showed that the negative slope of the basin reduces the sequent depth ratio, while a positive slope increases the sequent depth ratio. Gandhi [14] study the characteristics of supercritical flow in rectangular channel.

This paper presents the results of a study undertaken to determine the effects of bed slope, on the hydraulic jump characteristic.

2. THEORY

The momentum equation applied between sections 1 and 2 is written as follows:

$$\rho Qv_1 + P_1 + W \sin \alpha = \rho Qv_2 + P_2 \quad (1)$$

Where ρ is the density and Q is flow discharge. Fig. 1 shows a hydraulic jump evolving in a channel with positive slope. The following hypotheses will be considered in sections 1 and 2: that the pressure is hydrostatic and the friction forces are negligible. The weight W of the jump and the pressure forces P_1 and P_2 can be expressed by applying the hydrostatics laws as

$$P_1 = m\omega \frac{Y_1^3 \cos \theta}{3}; P_2 = m\omega \frac{Y_2^3 \cos \theta}{3}; W = \omega v \quad (2)$$

where Y_1 and Y_2 are flow depths, v_1 and v_2 are average velocities, θ is the angle of inclination of the channel, V is the volume of water included between sections 1 and 2; m is the mass of water included between section 1 and 2; and ω is the specific weight of the fluid. Geometrically, the volume V of the jump in a channel can be deduced from the quarter of a pyramid (Fig. 1).

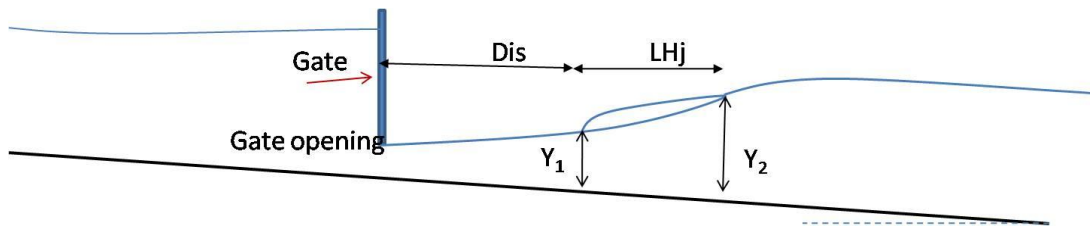


Fig. 1. Geometrical description of the volume of the jump

If Y_1 and v_1 are, respectively, the depth and mean velocity of the supercritical stream just upstream of the jump, with a Froude number of $Fr_1 = \frac{v_1}{\sqrt{gy_1}}$ where g is the acceleration due to gravity, the subcritical sequent depth Y_2 is given by the well-known Belanger equation Bélanger [15]

$$\frac{Y_2}{Y_1} = \frac{\sqrt{1+8Fr_1^2}-1}{2} \quad (3)$$

The total energy losses between section 1 and section 2 were calculated by using the following formula (Rajaratnam [4]):

$$E_L = \frac{H_3}{H_c} - \left(\frac{1+S_t}{2} \right) \times \left(\sqrt{1+8(Fr_1)^2} - 1 \right) + \frac{(Fr_1)^2}{2} \left(1 - \frac{4}{(1+S_t)^2 \times \left(\sqrt{1+8(Fr_1)^2} - 1 \right)^2} \right) \quad (4)$$

Where: S_t is the submergence ratio $(Y_t - Y_2)/Y_2$, Y_t is tail water depth, Y_2 is sequent water depth of the classical hydraulic jump.

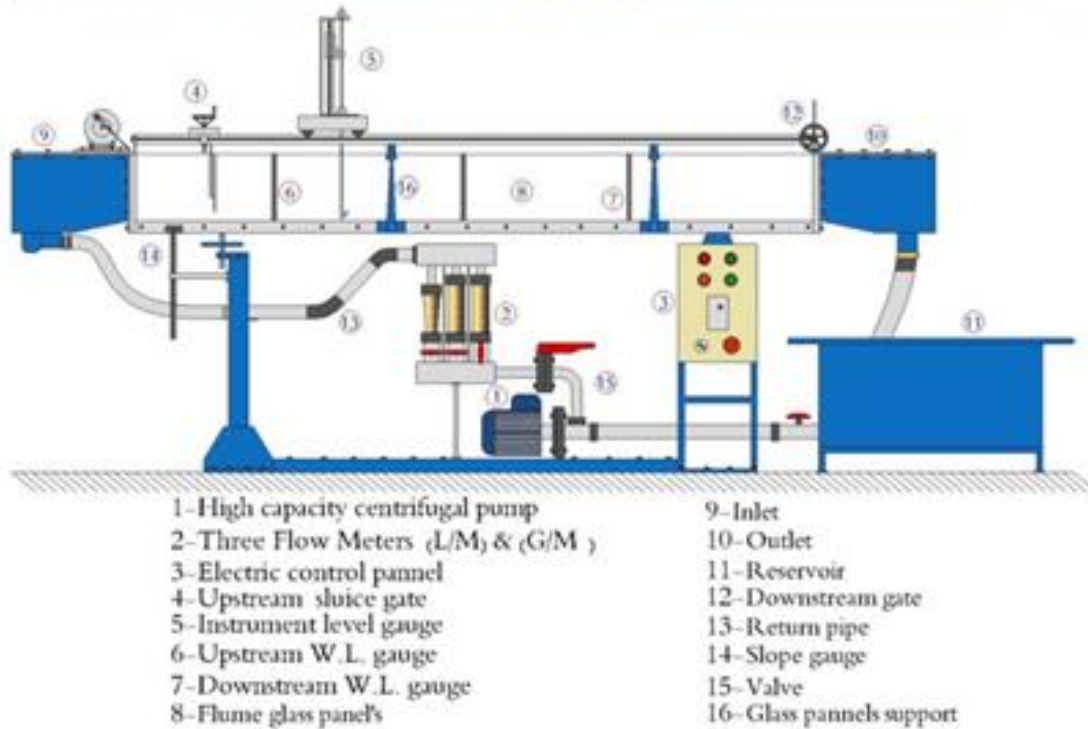
In addition, the relative energy losses $E_L/E_1 * 100$ was also calculated in order to check the efficiency of the design of new spillway stilling basin.

$$\Delta E = Y_1 + \frac{v_1^2}{2g} - Y_2 - \frac{v_2^2}{2g} \quad (5)$$

3. THE EXPERIMENTAL WORK

The experiments were conducted in a recirculation self contained tilting glass sided flume in the Hydraulics Laboratory of Faculty of

Engineering, Shoubra, Benha University. The flume is 2.5 m long, 9cm wide and 30 cm deep. A discharge control valve was used to regulate the flow rate. The bottom slope was adjusted using a screw jack located at the upstream end of the flume while at the downstream end; the flume was allowed to rotate freely about a hinged pivot. The slope was directly determined using a slope indicator. A downstream adjustable gate was used to regulate the tail-water surface elevation. The sidewalls along the entire length of the flume are made of glass with metal-frames, to allow visual investigation of the flow patterns. The horizontal bottom of the flume was made of steel and provided with a PVC pipe to drainage the water from the flume. The water entered the flume from an external water source, which was fed by an electric centrifugal pump. The water discharged the flume through two pumps with different capacities; 90, and 160 lit/min. Fig. 2 illustrates a complete and detailed flume description. A series of runs at different values of discharge were experimented and hydraulic jump was formed by operating the sluice gate and different discharge. For each run initial depth, sequent depth and length of hydraulic jump were measured. The experiments were carried out mainly by using five different gate opening, d_g of 1, 1.5, 2, 2.5, and 3 cm. Five positive bottom slopes, S_o were used. Five slopes (0.0027, 0.004, 0.0054, 0.0081, and 0.011) were used in the experimental. Four different flow rates ranged from 90 to 150 LPM were used for each particular conduit height and bottom slope. The initial Froude number ranged from 1.2 to 4.6. The discharge was measured using a pre-calibrated orifice meter. Depth measurements were taken using a point gauge with an accuracy of ± 0.1 mm. For each run, the initial depth of jump, the flow rate and the depth of water just downstream the conduit outlet were measured. A total of 80 experiments were conducted and the primary details of these experiments are shown in Table 1.



- | | |
|------------------------------------|--------------------------|
| 1-High capacity centrifugal pump | 9-Inlet |
| 2-Three Flow Meters (L/M) & (G/M) | 10-Outlet |
| 3-Electric control pannel | 11-Reservoir |
| 4-Upstream sluice gate | 12-Downstream gate |
| 5-Instrument level gauge | 13-Return pipe |
| 6-Upstream W.L. gauge | 14-Slope gauge |
| 7-Downstream W.L. gauge | 15-Valve |
| 8-Flume glass panels | 16-Glass pannels support |

Fig. 2. Schematic layout of experimental setup

Table 1. Primary details of experiments

Run no.	Slope (S)	Gate opening (dg) cm	Q (LPM)
I1	0.0027, 0.004, 0.0054, 0.0081, and 0.011	1	90
I2	0.0027, 0.004, 0.0054, 0.0081, and 0.011	1	120
I3	0.0027, 0.004, 0.0054, 0.0081, and 0.011	1	135
I4	0.0027, 0.004, 0.0054, 0.0081, and 0.011	1	150
II1	0.0027, 0.004, 0.0054, 0.0081, and 0.011	1.5	90
II2	0.0027, 0.004, 0.0054, 0.0081, and 0.011	1.5	120
II3	0.0027, 0.004, 0.0054, 0.0081, and 0.011	1.5	135
II4	0.0027, 0.004, 0.0054, 0.0081, and 0.011	1.5	150
III1	0.0027, 0.004, 0.0054, 0.0081, and 0.011	2	90
III2	0.0027, 0.004, 0.0054, 0.0081, and 0.011	2	120
III3	0.0027, 0.004, 0.0054, 0.0081, and 0.011	2	135
III4	0.0027, 0.004, 0.0054, 0.0081, and 0.011	2	150
IV1	0.0027, 0.004, 0.0054, 0.0081, and 0.011	2.5	90
IV2	0.0027, 0.004, 0.0054, 0.0081, and 0.011	2.5	120
IV3	0.0027, 0.004, 0.0054, 0.0081, and 0.011	2.5	135
IV4	0.0027, 0.004, 0.0054, 0.0081, and 0.011	2.5	150
V1	0.0027, 0.004, 0.0054, 0.0081, and 0.011	3	90
V2	0.0027, 0.004, 0.0054, 0.0081, and 0.011	3	120
V3	0.0027, 0.004, 0.0054, 0.0081, and 0.011	3	135
V4	0.0027, 0.004, 0.0054, 0.0081, and 0.011	3	150

4. RESULTS AND DISCUSSION

The deviation of different hydraulic jump characteristics such as sequent depth ratio, relative height of the jump, distance to Hydraulic jump, and relative length of the jump, relative energy loss with approach Froude number and slope of the channel is given below.

4.1 The Sequent Depth Ratio

The sequent depth ratio (Y_2/Y_1) was measured and plotted with respect to Froude number. Fig. 3 shows the variation of sequent depth ratio with approach Froude number and slope of the channel. It is observed that sequent depth ratio increases with increase in approach Froude number and slope of the channel bed. It is obvious from the figure that approximately 95% and 99.7% of experimental data are laying within $\pm 10\%$ of best fit curve. The increase of Froude number by 100% the relative sequent depth increases by 85%. At same Froude number the increase of channel slope causes increase of the relative depth. It is clear that the sequent depth ratio (Y_2/Y_1) increases with the increase of the slope at particular inlet Froude number. The

lowest value is due to the minimum slope (0.0027) and the maximum value due to the maximum slope (0.011).

Fig. 4 presents the variations of Y_2/Y_1 with the bottom slope (0.0027, 0.004, 0.0054, 0.0081, and 0.011) for different inlet Froude number. It is clear that the depth ratio increases with the increase of the slope at particular inlet Froude number. The lowest value is due to the minimum slope (0.0027) and the maximum value is due to the maximum slope (0.011). This could be explained by the fact that, the weight component acts in the direction of the flow. This in turn results in a reduction of the depth ratio and the rate of reduction increase as the slope decreases. The multiple linear regressions were applied to predict a statistical equation that correlates sequent depth ratio with other independent parameters (F_{r1} , S_o , Q , dg , and Y_1) in the form;

$$\frac{Y_2}{Y_1} = -0.058 + 1.935 * F_{r1} + 0.1657 * F_{r1}^2 - \frac{0.0026}{S_o} \quad (6)$$

With correlation coefficient equals 0.95.

$$\frac{Y_2}{Y_1} = \exp(489.74Q + 9.38S_0 - 71.8d_g + 1.57) \quad (7)$$

With correlation coefficient equals 0.90.

Where: Y_1 is the conjugate depth of flow U.S. the jump, Y_2 is the sequent depth of flow D.S. the jump, Q is the flow discharge, S_0 is the slope of the channel, d_g is the gate opening, and Fr_1 is the Froude number at vena contracta.

Fig. 5 shows the comparison of sequent depth ratio of present model equations 6, and 7 with the model equations developed by [4,16,17]. It is observed that most of the experimental data are lying between the present model and previous

model. It proves that the present model gives high accuracy.

4.2 Energy Dissipation

Fig. 6 shows the relation between the Froude number and relative energy loss (EL/E_1) for different slope. It is observed that relative energy loss increases non-linear with increase in approach Froude number from 1.2 to 4.6 and slope of the channel bed. It is evident from the figure that approximately 98%, 97%, 93% and 92% of experimental data are laying within $\pm 8\%$ of best fit curve.

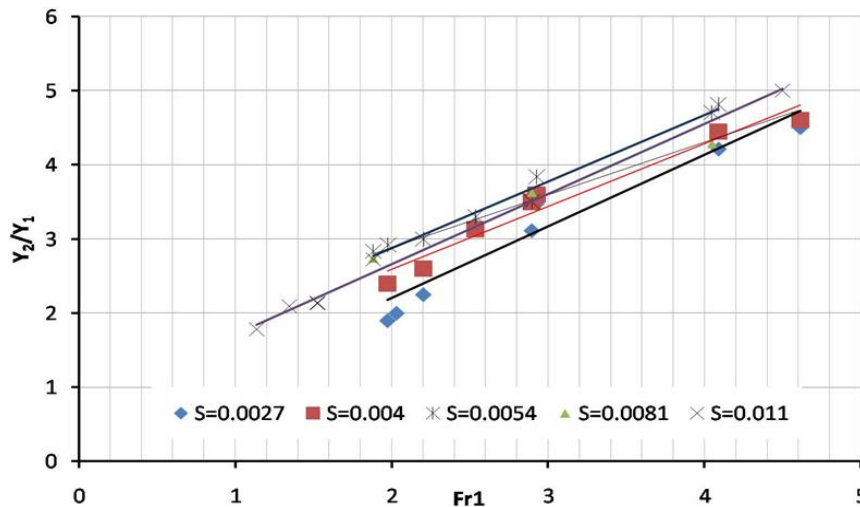


Fig. 3. Variation of sequent depth ratio with approach Froude number and channel slope

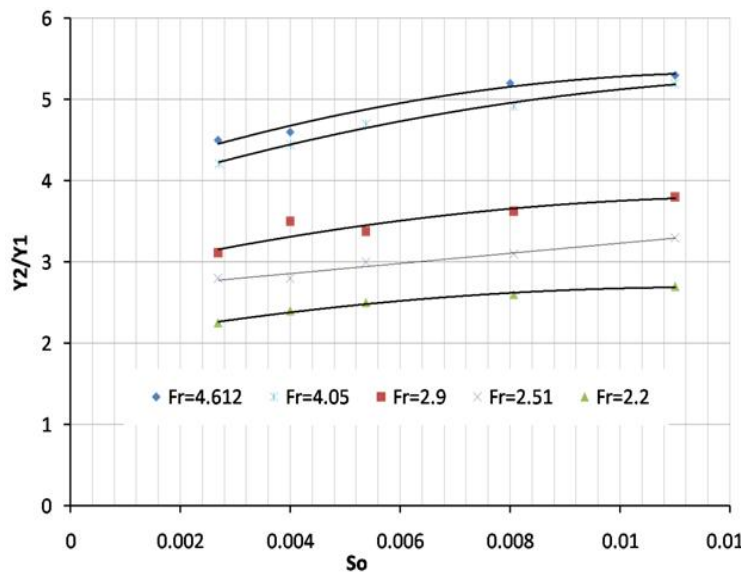


Fig. 4. Variations of Y_2/Y_1 with slope for different values of Froude number

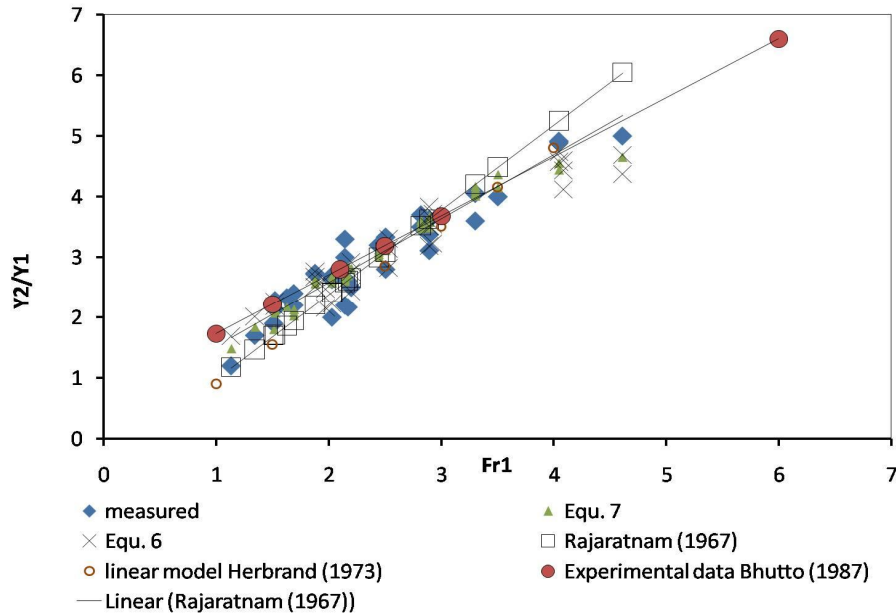


Fig. 5. The relation between the Froude number and the depth ratio

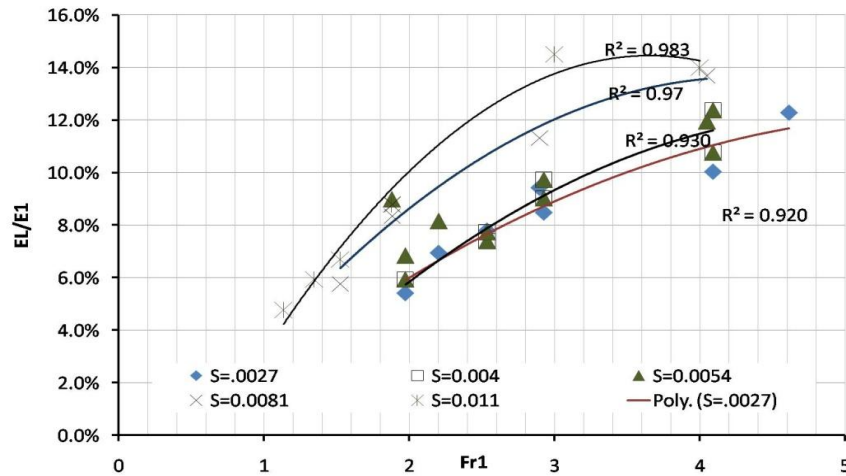


Fig. 6. Variation of relative energy loss with approach Froude number and channel slope

4.3 Length of Hydraulic Jump

Fig. 7 illustrates the relation between hydraulic jump length (L_{Hj}) and the sequent depth (Y_2) for different slopes. It clears that the hydraulic jump length increase by 80% as increase of sequent depth by 100%. For same sequent length the hydraulic jump length increase by decreases the slope of the channel bed. Fig. 8 shows the relation between the relative length of jump

(L_{Hj}/Y_1) and the Froude number Fr_1 for different slope. It is observed that relative length of the jump increases with increase in approach Froude number and slope of the channel bed.

In order to derive an equation with the corrected relative hydraulic jump length L_{Hj}/Y_1 as the dependent variable, the regression analysis tool of "Datafit ver. 9.0" was used to obtain the following equations:

$$L_{Hj}/Y_1 = 0.415 + 1.0315 \cdot Fr_1 - 0.0017/S_o \quad R^2 = 0.90 \quad (8)$$

$$L_{Hj}/Y_1 = 0.879(Y_2/Y_1) + 0.303 \quad R^2 = 0.927 \quad (9)$$

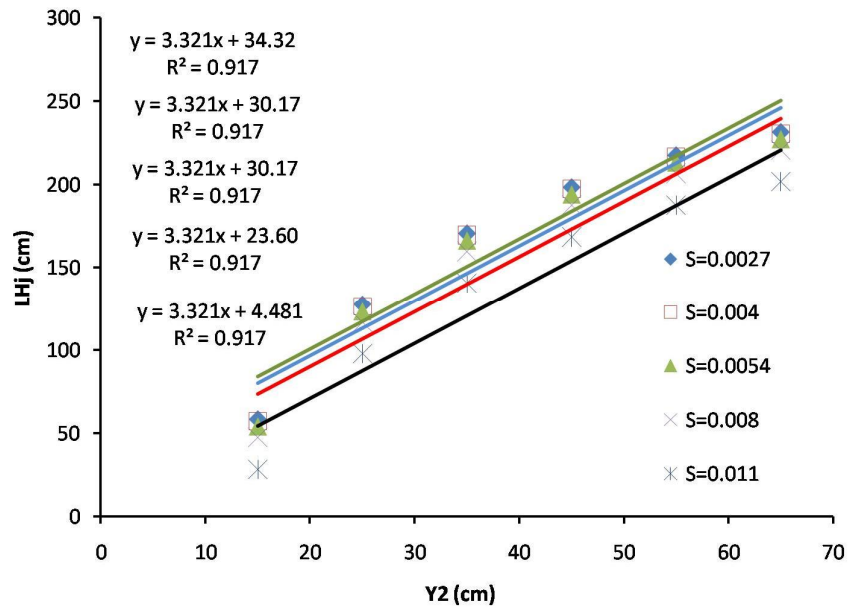


Fig. 7. The hydraulic jump length with approach sequent depth and channel slope

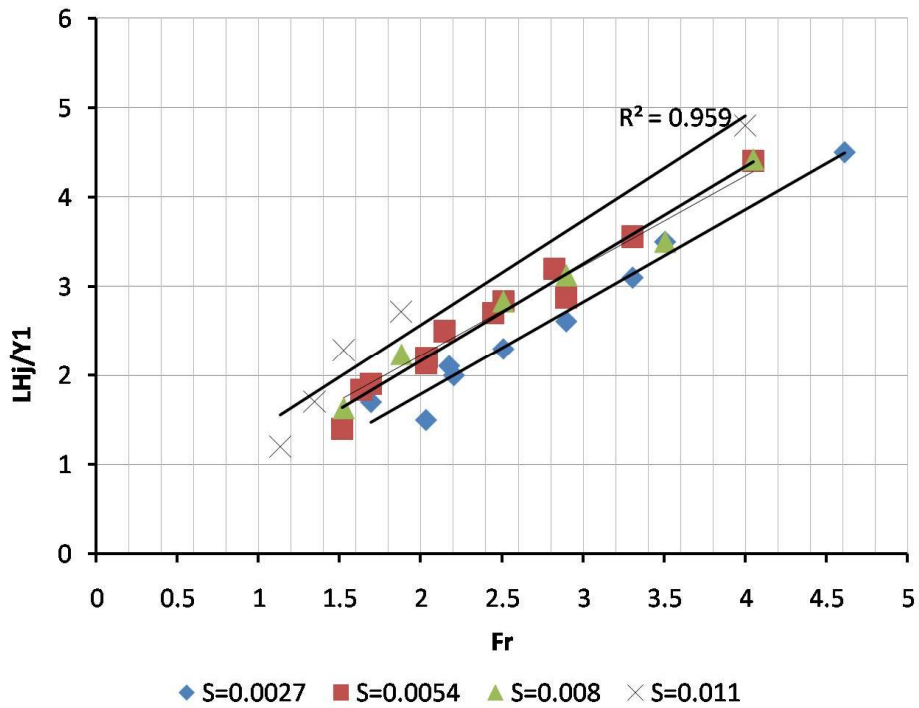


Fig. 8. The relative hydraulic jump length with approach Froude number and channel slope

Fig. 9 shows the comparison of relative height of the jump of present model equation 8 with the model equations developed by [18,16,17]. It is observed that most of the experimental data of Husain [16] are lying between the present model and Herbrand [17] model.

4.4 Distance to Hydraulic Jump

Fig. 10 shows the relation between the relative distance of jump (D_{is}/Y_1) and the Froude number

F_{r1} for different slope. It is observed that relative distance of the jump increases with increase in approach Froude number and slope of the channel bed. The multiple linear regressions were applied to predict a statistical equation that correlates D_{is}/Y_1 with other Froude number F_{r1} and different slope which represented by equation 9. Fig. 11 shows the comparison between equation 10 and the measured data. Clearly good agreement was achieved.

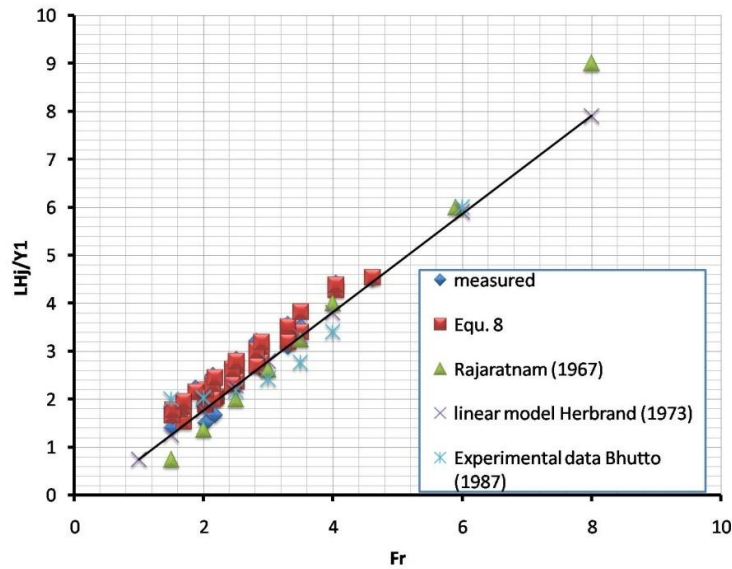


Fig. 9. The relative hydraulic jump length with approach Froude number and channel slope

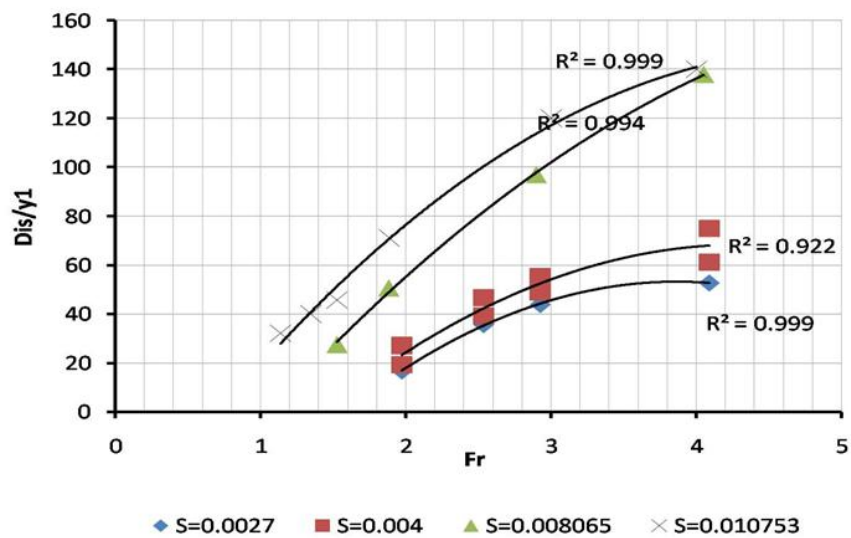


Fig. 10. Variation of relative slip length with approach Froude number and channel slope

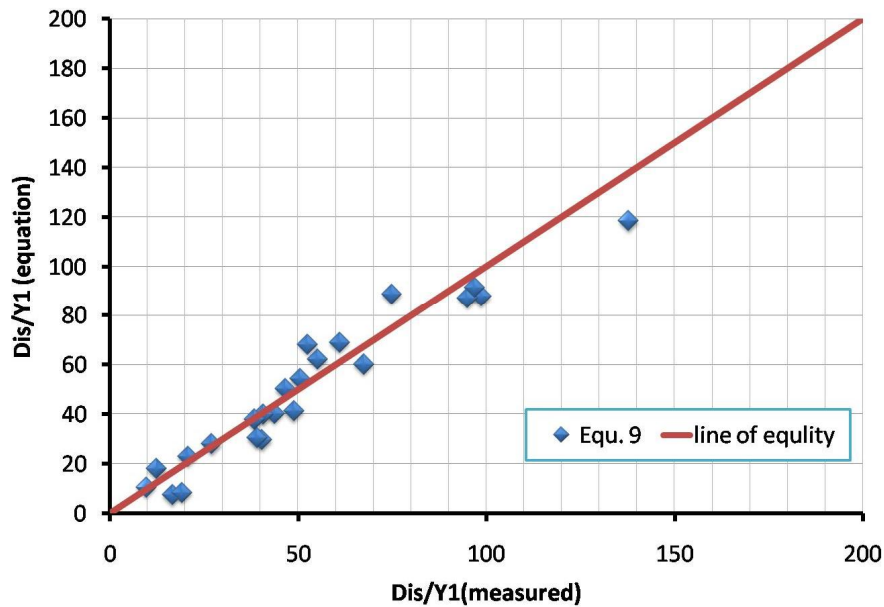


Fig. 11. Correlation between Dis/Y1 values, equation (10), and the corresponding observed ones

$$\frac{D_{is}}{Y_1} = -52.3 + 183.13 \cdot F_{r1} - 48.23 \cdot F_{r1}^2 + 4.81 \cdot F_{r1}^3 - \frac{1}{S_o} + \frac{0.0016}{S_o^2} \quad R^2 = 0.92 \quad (10)$$

This equation has a multi-linear correlation coefficient of 0.92. The computed values of D_{is}/Y_1 using the above equation are plotted against the corresponding experimental data as shown in Fig. (11).

5. CONCLUSION

Hydraulic jumps produced in rectangular channel with different slopes and different Froude numbers were analyzed. It was found that the bottom slope "So", and the inlet Froude number Fr_1 , have major effect on the variations of the jump outlet characteristics. The sequent depth ratio increases with increase in approach Froude number and slope of the channel bed. The increase of Froude number by 100% the relative sequent depth increases by 85%. At same Froude number the increase of channel slope causes increase of the relative depth. The relative energy loss increases non-linear with increase Froude number and slope of the channel bed. The hydraulic jump length increases by 80% as increase of sequent depth by 100%. For same sequent length the hydraulic jump length increase by decreases the slope of the channel bed. The relative length of the jump increases with increase in approach Froude

number and slope of the channel bed. The relative distance of the jump increases with increase in approach Froude number and slope of the channel bed. The prediction of the projected model was compared to the previously developed models. The predicted results agreed well with the experimental observations as well as with those of the before developed models using the same technique. The developed experimental computational models are applicable between Froude numbers from 1.2 to 4.6.

COMPETING INTERESTS

Author has declared that no competing interests exist.

REFERENCES

1. Chern M, Syamsuri S. Effect of corrugated bed on hydraulic jump characteristic using SPH method. *J. Hydraul. Eng.* 2013; 139(2):221–232.
2. Kordi E, Abustan I. Transitional expanding hydraulic jump. *J. Hydraul. Eng.* 2012; 138(1):105–110.

3. Peterka AJ. Hydraulic design of stilling basins and energy dissipaters. Engineering Monograph No. 25, U.S. Bureau of Reclamation, Denver Colorado; 1958.
4. Rajaratnam N. Hydraulic jumps. Adv. Hydro. Sci. 1967;4:197-280.
5. McCorquodale JA. Hydraulic jumps and internal flows Chapter 8. In: Encyclopedia of Fluid Mechanics, Cheremisionoff NP. (Ed.). Vol. 2. Gulf Publishing, Houston. 1986;120-173.
6. Hager W. Impact hydraulic jump. J. Hydraul. Eng. 1994;120(5):633–637.
7. Hager WH. Energy dissipaters and hydraulic jump. Kluwer Academic, Dordrecht; 1992.
8. Hughes W, Flack J. Hydraulic jump properties over a rough bed. J. Hydraul. Eng. 1984;110(12):1755–1771.
9. Afzal N, Bushra A, Seena A. Analysis of turbulent hydraulic jump over a transitional rough bed of a rectangular channel: Universal relations. J. Eng. Mech. 2011; 137(12):835–845.
10. Abdel-Azim M Negm. Analysis and formulation of hydraulic in sloping rectangular closed conduits. Seventh International Water Technology Conference Egypt; 2003.
11. Chyan-Deng Jan, Chia-Jung Chang. Hydraulic jump in an inclined rectangular chute contraction. Journal of Hydraulic Engineering, ASCE. DOI 10.1061, Vol. 135, No 11, ISSN 0733- 9429/2009/11-949-958; 2009.
12. Smith CD, Wentao Chen. The hydraulic jump in a steeply sloping square conduit. Journal of Hydraulic Research (Impact Factor: 1.35). 01/2010. 2010;1989:385-399.
13. Beirami M, Chamani M. Hydraulic jumps in sloping channels: Sequent depth ratio. J. Hydraul. Eng. 2006;132(10):1061–1068.
14. Gandhi S, Yadav V. Characteristics of supercritical flow in rectangular channel. International Journal of Physical Sciences. 2013;8(40):1934-1943.
15. Bélanger JB. Notes sur le cours dhydraulique Notes on a course in hydraulics. Mém. Ecole Nat. Ponts et Chaussées, Paris, France, session 1849–1850, 222 pages in French; 1849.
16. Husain Baksh G Bhutto. Hydraulic jump control and energy dissipation. PhD Thesis. Department of Hydraulics and Irrigation Engineering, Mehran Institute of Engineering and Technology, Jamshoro; 1987.
17. Herbrand K. The spatial hydraulic jump. Journal of Hydraulic Research. 1973;11(3): 205–218.
18. Rajaratnam N, Subramanya K. Profile of hydraulic jump. Journal of Hydraulic Division. ASCE. 1968;94(3):663–673.

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