Some Common Fuzzy Fixed Point Theorems on G - Metric Space

P. Senthil Kumar [a] and P. Thiruveni [a*]

[a] *Department of Mathematics, Rajah Serfoji Government College (Autonomous) (Affiliated To Bharathidasan University), Thanjavur, India.*

Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

Original Research Article

Abstract

The motive of this paper is to give a few not unusual place constant factor theorems in G-Metric spaces, with the aid of using the perception of Common restrict with inside the variety belongings and to illustrate appropriate examples. These outcomes make bigger and generalizes numerous widely known outcomes with inside the literature.

_____

*Corresponding author: Email: smile.thiruveni@gmail.com;

1 Introduction

Inspired through the truth that metric fixed point idea has a huge utility in nearly all fields of quantitative sciences, many authors have directed their interest to generalize the belief of a metric space. In this respect, several generalized metric spaces have come thru through many authors, with inside the remaining decade. Among all of the generalized metric areas, the belief of G-Metric space has attracted substantial interest from fixed point theorists. The idea of a G-Metric space become brought through Mustafa and Sims in [1], in which the authors mentioned the topological residences of this area and proved the analog of the Banach contraction precept with inside the context of G-metric spaces. Following those outcomes, many authors have studied and advanced numerous not unusual place fixed point theorems on this framework. In 2002, M.Aamri and D.El Moutawakil [2] brought the belongings (E.A), which is a real generalization of non-compatible maps in metric spaces. Under this belief many not unusual place constant factor theorems have been studied with inside the literature (see [3-8] and the references therein). The idea of Common restriction with inside the variety of g (CLRg) belongings for a couple of self mappings in Fuzzy metric area [9-13]. The significance of this belongings is, it guarantees that one does now no longer require the closeness of the variety subspaces and hence, now a days, authors are giving an awful lot interest to this belongings for generalizing the outcomes gift with inside the literature(see [12] and the references therein). Very recently, this become prolonged to 2 pairs of self mappings as CLR (S,T) belongings. In the existing paper, through using the notions of not unusual place restriction with inside the variety belongings for 2 in addition to 4 self maps and susceptible compatibility, which is a good device in offering the not unusual place fixed points, we derive a few not unusual place fixed point theorems with inside the realm of G-metric space, which generalizes diverse similar outcomes in [4] and others. At the equal time, we gift appropriate examples to showcase the application of the principle outcomes presented. The following are the primary definitions wanted with inside the primary outcomes.

1.1 Observation

Definition 1.1:

The 3-tuple $(\mathbb{X}, \mathbb{M}, *)$ is called a fuzzy metric space if $\mathbb{X}$ is an arbitrary (non-empty) set, $*$ is a continuous t-norm and $\mathbb{M}$ is a fuzzy set on $\mathbb{X}^2 \times [0, \infty)$ satisfying the following conditions, for all $a, b, c \in \mathbb{X}$, each s and $t > 0$

(i) $\mathbb{M}(a, b, 0) > 0$
(ii) $\mathbb{M}(a, b, t) = 1$ if and only if $a = b$,
(iii) $\mathbb{M}(a, b, t) = \mathbb{M}(b, a, t)$,
(iv) $\mathbb{M}(a, b, t) * \mathbb{M}(b, c, u) \leq \mathbb{M}(a, b, t + u)$,
(v) $\mathbb{M}(a, b, \diamond) : (0, \infty) \to [0,1]$ is left continuous.

If, in the above definition, the triangular inequality (iv) is replaced by

$\mathbb{M}(a, b, t) * \mathbb{M}(b, c, u) \leq \mathbb{M}(a, b, \max\{t, u\})$ for all $a, b, c \in \mathbb{X}$, $t, u > 0$, or equivalently,

$$\mathbb{M}(a, b, t) * \mathbb{M}(b, c, t) \leq \mathbb{M}(a, c, t)$$

and then the triple $(\mathbb{X}, \mathbb{M}, *)$ is called a non - Archimediean fuzzy metric space.

Definition 1.2:

Let $\mathbb{X}$ be a non empty set and let $\mathcal{G} : \mathbb{X} \times \mathbb{X} \times \mathbb{X} \to [0, \infty)$ be a function satisfying the following properties

(i) $\mathcal{G}(a, b, c) = 0$ if $a = b = c$.
(ii) $0 < \mathcal{G}(a, a, b)$ for all $a, b \in \mathbb{X}$ with $a \neq b$.
(iii) $\mathcal{G}(a, a, b) \leq \mathcal{G}(a, b, c)$ for all $a, b, c \in \mathbb{X}$ with $b \neq c$.
(iv) $\mathcal{G}(a, b, c) = \mathcal{G}(a, c, b) = \mathcal{G}(b, c, a) = \cdots \ldots \ldots$, symmetry all three variables.
(v) $\mathcal{G}(a, b, c) \leq \mathcal{G}(a, x, x) + \mathcal{G}(x, b, c)$ for all $a, b, c, x \in \mathbb{X}$.

Then the function $\mathcal{G}$ is called a generalized metric or a $\mathcal{G}$ - metric on $\mathbb{X}$ and the pair $(\mathbb{X}, \mathcal{G})$ is called a $\mathcal{G}$ – metric space.

Example 1.1: Let $(\mathbb{X}, d)$ be the usual metric space Then the function

$$\mathcal{G}: \mathbb{X} \times \mathbb{X} \times \mathbb{X} \to [0, \infty) \text{ defined by } \mathcal{G}(a, b, c) = max\{d(a, b), d(b, c), d(c, a)\}$$

for all $a, b, c \in \mathbb{X}$, is a $\mathcal{G}$ -Metric space.

In their initial paper Mustafa and Sims [3] also proved the following proposition.

Proposition: Let $(\mathbb{X}, \mathcal{G})$ be a $\mathcal{G}$ -metric space. Then, for any a, b, c, x, y, z $\in \mathbb{X}$, it follows that

1. if $\mathcal{G}(a, b, c) = 0$, then a = b = c
2. $\mathcal{G}(a, b, c) \leq \mathcal{G}(a, a, b) + \mathcal{G}(a, a, c)$
3. $\mathcal{G}(a, b, b) \leq 2\mathcal{G}(b, a, a)$.
4. $\mathcal{G}(a, b, c) \leq \mathcal{G}(a, x, c) + \mathcal{G}(x, b, c)$.
5. $\mathcal{G}(a, b, c) \leq \frac{2}{3}\{\mathcal{G}(a, b, x) + \mathcal{G}(a, x, c) + \mathcal{G}(x, b, c)\}$.
6. $\mathcal{G}(a, b, c) \leq \mathcal{G}(a, x, x) + \mathcal{G}(b, x, x) + \mathcal{G}(c, x, x)$.

Definition 1.3:

The G- metric space $(\mathbb{X}, \mathcal{G})$ is called symmetry if $\mathcal{G}(a, a, b) = \mathcal{G}(a, b, b)$ for all $a, b \in \mathbb{X}$.

Definition 1.4:

A 3 – tuple $(\mathbb{X}, \mathcal{G}, *)$ is called a $\mathcal{G}$ – fuzzy metric space if $\mathbb{X}$ is an arbitrary non empty set, $*$ is a continuous t-norm and $\mathcal{G}$ is a Fuzzy set on $\mathbb{X}^3 \times (0, \infty)$ satisfying the following conditions for each $t, s > 0$.

   (i) $\mathcal{G}(a, a, b, t) > 0$ for all a, b $\in \mathbb{X}$ with a $\neq$ b.
   (ii) $\mathcal{G}(a, a, b, t) \geq \mathcal{G}(a, b, c, t)$ for all a, b, c $\in \mathbb{X}$ with b $\neq$ c.
   (iii) $\mathcal{G}(a, b, c, t) = 1$ if and only if $a = b = c$.
   (iv) $\mathcal{G}(a, b, c, t) = \mathcal{G}(p(a, b, c), t)$, where p is a permutation function.
   (v) $\mathcal{G}(a, b, c, t + s) \geq \mathcal{G}(x, b, c, t) * \mathcal{G}(a, x, x, s)$ for all $a, b, c, x \in \mathbb{X}$.
   (vi) $\mathcal{G}(a, b, c, .): (0, \infty) \to [0,1]$ is continuous.

Definition 1.5:

A $\mathcal{G}$ - fuzzy metric space $(\mathbb{X}, \mathcal{G}, *)$ is said to be symmetric if $\mathcal{G}(a, a, b, t) = \mathcal{G}(a, b, b, t)$ for all $a, b \in \mathbb{X}$ and for each $t > 0$.

Definition 1.6:

A pair of self – mappings $(g, \hbar)$ of a fuzzy metric spaces $(\mathbb{X}, \mathcal{M}, *)$ is said to satisfy E.A property, if there exists a sequence $\{a_n\}$ in $\mathbb{X}$ such that $\lim_{n \to \infty} \mathcal{M}(fa_n, ga_n, t) = 1$ for some $t \in \mathbb{X}$.

Definition 1.7:

Let $g, \hbar, \mathcal{K}, \mathcal{L}$ be four self mappings defined on a symmet ric space $(\mathbb{X}, d)$. Then the pairs $(g, \mathcal{K})$ and $(\hbar, \mathcal{L})$ are said to have the common limit range property (with respect to $\mathcal{K}$ and $\mathcal{L}$) often denoted by $CLR_{(\mathcal{K}, \mathcal{L})}$ property, if there exists two sequences $\{x_n\}$ and $\{y_n\}$ in $\mathbb{X}$ such that

$$lim_n \, gx_n = lim_n \, \mathcal{K}x_n = lim_n \, gy_n = lim_n \, \mathcal{L}x_n = t \text{ with}$$

$$t = \mathcal{K}u = \mathcal{L}w \text{ for some } t, u, w \in \mathbb{X}.$$

If $g = \hbar$ and $\mathcal{K} = \mathcal{L}$, then the above definition implies $CLR_{(\mathcal{K},\mathcal{L})}$ property due to Sintunavarat et al.

2 Main Result

The first end result is a not unusual place fuzzy constant factor theorem for a couple of self mappings the usage of a generalized strict contractive condition, which expand Theorem 1 of [4].

Theorem 2.1:

Let $(\mathbb{X}, \mathcal{G})$ be a symmetric $\mathcal{G}$ - Metric space and f, g be two weakly compatible self mappings on $\mathbb{X}$ satisfying [14]

1. $CLR_{(\mathcal{K},\mathcal{L})}$ property.
2. $\mathcal{G}(ga, gb, gc, kt) <$
   $\max \{\mathcal{G}(\hbar a, \hbar b, \hbar c), \frac{\mathcal{G}(ga,\hbar a,\hbar a,t)+\mathcal{G}(gb,\hbar b,\hbar b,t)+\mathcal{G}(gc,\hbar c,\hbar c,t)}{3}, \frac{\mathcal{G}(gb,\hbar a,\hbar a,t)+\mathcal{G}(gc,\hbar b,\hbar b,t)+\mathcal{G}(ga,\hbar c,\hbar c,t)}{3}\}$ For all
   $a, b, c \in \mathbb{X}$ with $a \neq b$.

Then g and $\hbar$ have a unique common fuzzy fixed point.

Proof:

Since g and $\hbar$ satisfies $CLR_{(\mathcal{K},\mathcal{L})}$ property, there exists a sequence

$\{x_n\}$ in $\mathbb{X}$ such that $lim_n \, gx_n = lim_n \, \hbar x_n = \hbar x$ for some $x \in \mathbb{X}$.

Consider
$\mathcal{G}(gx_n, ga, ga, kt) <$
$\max \{\mathcal{G}(\hbar x_n, \hbar a, \hbar a), \frac{\mathcal{G}(gx_n,\hbar x_n,\hbar x_n,t)+\mathcal{G}(ga,\hbar a,\hbar a,t)+\mathcal{G}(ga,\hbar a,\hbar a,t)}{3}, \frac{\mathcal{G}(ga,\hbar x_n,\hbar x_n,t)+\mathcal{G}(ga,\hbar a,\hbar a,t)+\mathcal{G}(ga,\hbar c,\hbar c,t)}{3}\}$

Letting n $\rightarrow \infty$, we obtain $\mathcal{G}(\hbar a, ga, ga) \leq \frac{2}{3} \mathcal{G}(\hbar a, ga, ga)$ which implies

$\hbar a = ga$.

Thus x is the coincidence point of g and $\hbar$. Let $c = ga = \hbar a$.

Since $(g, \hbar)$ are weakly compatible, we have $gc = g\hbar c = \hbar gc = \hbar c$.

Now we will prove that $gc = c$.

Suppose $gc \neq c$, then

$\mathcal{G}(gc, c, c, kt) = \mathcal{G}(gc, ga, ga, kt) <$
$max\{\mathcal{G}(gc, ga, ga, t), \frac{\mathcal{G}(ga,\hbar a,\hbar a,t)+\mathcal{G}(gb,\hbar b,\hbar b,t)+\mathcal{G}(gc,\hbar c,\hbar c,t)}{3}, \frac{\mathcal{G}(gb,\hbar a,\hbar a,t)+\mathcal{G}(gc,\hbar b,\hbar b,t)+\mathcal{G}(ga,\hbar c,\hbar c,t)}{3}\}$

$< \mathcal{G}(gc, c, c, t)$, which is a contradiction.

Hence $gc = c = \hbar c$.

Thus c is the common fixed point of g and $\hbar$.

The uniqueness of the fixed point can be proved easily.

We now illustrate this theorem by giving an example.

Example 2.1:

Let $\mathbb{X} = [2, 20]$ and $\mathcal{G}: \mathbb{X} \times \mathbb{X} \times \mathbb{X} \times (0, \infty) \longrightarrow [0, \infty)$ defined by $\mathcal{G}(a, b, c, t) = 0$ if $a = b = c$ and $\mathcal{G}(a, b, c, t) = max\{a, b, c\}$ in all other cases.

Then $(\mathbb{X}, \mathcal{G}, *)$ is a symmetric $\mathcal{G}$ - Metric space.

Let $g, \hbar$ be two self maps on $\mathbb{X}$ defined by,

$$gx = 5 \text{ if } x \leq 5, gx = 3 \text{ if } x > 5 \quad \text{and}$$
$$\hbar x = \frac{x+5}{2} \text{ if } x \leq 5, \hbar x = 10 \text{ if } x > 5.$$

Here g and $\hbar$ satisfies the $CLR_{\hbar}$ property.

To see this, consider a sequence

$$\{x_n\} = \left\{5 - \frac{1}{n}\right\} \text{ for all } n.$$

Then $gx_n = g\left(5 - \frac{1}{n}\right) \longrightarrow 5$ and $\hbar x_n = \hbar\left(5 - \frac{1}{n}\right) = \frac{5 - \frac{1}{n} + 5}{2} \longrightarrow 5$

Therefore $lim_n gx_n = lim_n \hbar x_n = 5 = \hbar 5$.

Further, $(g, \hbar)$ are weakly compatible and

$$\mathcal{G}(ga, gb, gc, \hbar t) < max\{\mathcal{G}(\hbar a, \hbar b, \hbar c, t), \frac{\mathcal{G}(ga, \hbar a, \hbar a, t) + \mathcal{G}(gb, \hbar b, \hbar b, t) + \mathcal{G}(gc, \hbar c, \hbar c, t)}{3},$$
$$\frac{\mathcal{G}(gb, \hbar a, \hbar a, t) + \mathcal{G}(gc, \hbar b, \hbar b, t) + \mathcal{G}(ga, \hbar c, \hbar c, t)}{3}\}$$

Thus g and $\hbar$ satisfy all the conditions of Theorem 2.1 and have a unique

Common fuzzy fixed point at $x = 5$.

In 1977, Mathkowski [5] introduced the Φ-map as the following:

Let Φ be the set of auxiliary functions φ such that $\varphi : [0, \infty) \longrightarrow [0, \infty)$ is a non-decreasing function satisfying $lim_n \varphi^n(t) = 0$ for all $t \in (0, \infty)$. If $\varphi \in \Phi$, then φ is called a Φ −map. further $\varphi(t) < t$ for all $t \in (0, \infty)$.

If $\varphi(0) = 0$.

In the next result, we extracted a unique common fixed point for two pairs of self mappings which involves a φ-map under the lipschitz type of contractive condition. This result extend and generalize Theorem 2 of M.Aamri and El Moutawakil [2].

Theorem 2.2:

Let $(\mathbb{X}, \mathcal{G}, *)$ be a symmetric fuzzy $\mathcal{G}$-Metric space and $g, \hbar, \mathcal{K}, \mathcal{L}$ be four self mappings on X such that

1. $(g, \mathcal{K})$ and $(\hbar, \mathcal{L})$ satisfies $CLR_{(\mathcal{K}, \mathcal{L})}$ property.
2. $\mathcal{G}(ga, \hbar b, \hbar c, \hbar t) \leq \varphi(max\{\mathcal{G}(\mathcal{K}a, \mathcal{L}b, \mathcal{L}c, t), \mathcal{G}(\mathcal{K}a, \mathcal{L}b, \hbar c, t), \mathcal{G}(\mathcal{L}b, \hbar b, \hbar c, t),$
   $\mathcal{G}(\hbar b, \mathcal{L}b, \mathcal{L}c, t)\}) \ \forall \ a, b, c \in \mathbb{X}$.
3. $(g, \mathcal{K})$ and $(\hbar, \mathcal{L})$ are weakly compatible.

Then $g, \hbar, \mathcal{K}$ and $\mathcal{L}$ have a unique fuzzy common fixed point.

Proof:

Since $(\mathcal{g}, \mathcal{K})$ and $(\hbar, \mathcal{L})$ satiesfies $\text{CLR}_{(\mathcal{K},\mathcal{L})}$ property, there exists two sequences

$\{x_n\}$ and $\{y_n\}$ such that $lim_n \, \mathcal{g}x_n = lim_n \, \mathcal{L}x_n = lim_n \, \hbar x_n = lim_n \, \mathcal{L}x_n = t$ with $t = \mathcal{K}x = \mathcal{L}y$ for some $t, x, y \in \mathbb{X}$.

Consider,

$$\mathcal{G}(\mathcal{g}a, \hbar y_n, \hbar y_n, kt)$$
$$< \varphi(\max\{\mathcal{G}(\mathcal{K}a, \mathcal{L}y_n, \mathcal{L}y_n, t), \mathcal{G}(\mathcal{K}a, \hbar y_n, \hbar y_n, t), \mathcal{G}(\mathcal{L}y_n, \hbar y_n, \hbar y_n, t), \mathcal{G}(\hbar y_n, \mathcal{L}y_n, \mathcal{L}y_n, t)\}\}$$

On letting $n \to \infty$, we obtain $\mathcal{G}(\mathcal{g}x, u, u, t) \leq \varphi(0) = 0$ which implies $\mathcal{g}x = u = \mathcal{K}x$.

Hence x is the coincidence point of $\mathcal{g}$ and $\mathcal{K}$. Since $(\mathcal{g}, \mathcal{K})$ are weakly compatible, we have $\mathcal{g}\mathcal{g}x = \mathcal{g}\mathcal{K}x = \mathcal{K}\mathcal{g}x = \mathcal{K}\mathcal{K}x$.

Now we prove that $\mathcal{L}b = \hbar b$.

Consider

$$\mathcal{G}(\mathcal{g}x, \hbar y, \hbar y, kt) <$$
$$\varphi(\max\{\mathcal{G}(\mathcal{K}x_n, \mathcal{L}b, \mathcal{L}b, t), \mathcal{G}(\mathcal{K}a, \hbar y_n, \hbar y_n, t), \mathcal{G}(\mathcal{L}y_n, \hbar y_n, \hbar y_n, t), \mathcal{G}(\hbar y_n, \mathcal{L}y_n, \mathcal{L}y_n, t)\})$$

On letting $n \to \infty$, we have

$$\mathcal{G}(u, \hbar b, \hbar b, t) \leq \varphi(\mathcal{G}(u, \hbar b, \hbar b, t)) \text{ which implies } \mathcal{G}(u, \hbar b, \hbar b, t) = 0.$$

Therefore $\hbar b = u = \mathcal{L}b$. that is b is the coincidence point of $\hbar$ and $\mathcal{L}$.

Since $(\hbar, \mathcal{L})$ are weakly compatible, we prove that $\hbar\hbar b = \hbar\mathcal{L}b = \mathcal{L}\hbar b = \mathcal{L}\mathcal{L}b$.

Also note that $\mathcal{g}a = \mathcal{K}a = \hbar b = \mathcal{L}b = u$.

Now we prove that $\mathcal{g}\mathcal{g}a = \mathcal{g}a$. suppose $\mathcal{g}a \neq \mathcal{g}\mathcal{g}a$, then

$$\mathcal{G}(\mathcal{g}\mathcal{g}a, \mathcal{g}a, \mathcal{g}a, kt) = \mathcal{G}(\mathcal{g}\mathcal{g}a, \hbar b, \hbar b, t)$$
$$\leq \varphi(\max\{\mathcal{G}(\mathcal{K}\mathcal{g}a, \mathcal{L}b, \mathcal{L}b, t), \mathcal{G}(\mathcal{K}\mathcal{g}a, \hbar b, \hbar b, t), \mathcal{G}(\mathcal{K}b, \hbar b, \hbar b, t), \mathcal{G}(\hbar b, \mathcal{L}b, \mathcal{L}b, t)\})$$
$$< \mathcal{G}(\mathcal{g}\mathcal{g}a, \mathcal{g}a, \mathcal{g}a, t) \text{ a contradiction.}$$

Hence $\mathcal{g}\mathcal{g}a = \mathcal{g}a = \mathcal{K}\mathcal{g}a$, which implies $\mathcal{g}a$ is the common fuzzy fixed point of $\mathcal{g}$ and $\mathcal{K}$.

Similarly one can prove $\hbar b$ is the common fuzzy fixed point of $\mathcal{g}$ and $\mathcal{L}$.

Since $\mathcal{g}a = \hbar b, c = \mathcal{g}a$ is the common fuzzy fixed point of $\mathcal{g}, \hbar, \mathcal{K}, and \, \mathcal{L}$.

The uniqueness of the fuzzy fixed point follows easily.

Corollary 2.1:

Let $(\mathbb{X}, \mathcal{G}, *)$ be a symmetric fuzzy $\mathcal{G}$-Metric space and $\mathcal{g}, \hbar, \mathcal{K}, \mathcal{L}$ be four self mappings on X such that

1. $(\mathcal{g}, \mathcal{K})$ and $(\hbar, \mathcal{L})$ satisfies $\text{CLR}_{(\mathcal{K},\mathcal{L})}$ property.
2. $\mathcal{G}(\mathcal{g}a, \hbar b, \hbar c, kt) \leq \varphi(\max\{\mathcal{G}(\mathcal{K}a, \mathcal{L}b, \mathcal{L}b, t), \mathcal{G}(\mathcal{K}a, \hbar b, \hbar b, t), \mathcal{G}(\mathcal{L}b, \hbar b, \hbar b, t),$
   $\mathcal{G}(\hbar b, \mathcal{L}b, \mathcal{L}b, t)\}) \, \forall \, a, b, c \in \mathbb{X}$.
3. $(\mathcal{g}, \mathcal{K})$ and $(\hbar, \mathcal{L})$ are weakly compatible.

Then g, $\hbar$, $\mathcal{K}$ and $\mathscr{L}$ have a unique fuzzy common fixed point.

Proof:

By restricting $g, \hbar, , \mathcal{K}, \mathscr{L}$ suitly, one can derive the corollaries involving two as well as three self mappings as follows:

Corollary 2.2:

Let $(\mathbb{X}, \mathcal{G}, *)$ be a symmetric $\mathcal{G}$ − metric space and $g, \hbar, \mathcal{K}$ be three self mappings on $\mathbb{X}$ such that

1. $(g, \mathcal{K})$ and $(\hbar, \mathscr{L})$ satisfies $\text{CLR}_{\mathcal{K}}$ property.
2. $\mathcal{G}(ga, \hbar b, \hbar c, kt) \leq \varphi(max\{\mathcal{G}(\mathcal{K}a, \mathcal{K}b, \mathcal{K}c, t), \mathcal{G}(\mathcal{K}a, \hbar b, \hbar c, t), \mathcal{G}(\mathscr{L}b, \hbar b, \hbar c, t), \mathcal{G}(\hbar b, \mathscr{L}b, \mathscr{L}c, t)\}) \ \forall \ a, b, c \in \mathbb{X}$.
3. $(g, \mathcal{K})$ and $(\hbar, \mathcal{K})$ are weakly compatible.

Then g, $\hbar$ and $\mathcal{K}$ have a unique fuzzy common fixed point.

Proof:

Follows from theorem 2.2 by setting $\mathcal{K} = \mathscr{L}$.

We now exhibit this theorem through the subsequent example.

Example 2.2:

Let $\mathbb{X} = [0,6]$ and $\mathcal{G}: \mathbb{X} \times \mathbb{X} \times \mathbb{X} \times (0, \infty) \longrightarrow [0, \infty)$ defined by $\mathcal{G}(a, b, c, t) = 0$ if $a = b = c$ and $\mathcal{G}(a, b, c, t) = max\{a, b, c\}$ in all other cases.

Then clearly $(\mathbb{X}, \mathcal{G}, *)$ is a symmetric $\mathcal{G}$ - Metric space.

Let $g, \hbar, \mathcal{K}, \mathscr{L}$ be four self maps on $\mathbb{X}$ defined by,

$$ga = 3 \ if \ a \leq 3, ga = 4 \ if \ a > 3 \quad and$$
$$\mathcal{K}a = 6 - a \ if \ a \leq 3, \mathcal{K}a = 5 \ if \ a > 3,$$

$$\hbar a = 4 \ if \ a < 3,$$
$$\mathscr{L}a = \frac{2a+3}{3} if \ x \geq 3$$

Now $(g, \mathcal{K})$ and $(\hbar, \mathscr{L})$ satisfy the $\text{CLR}_{(\mathcal{K}, \mathscr{L})}$ property.

To see this, choose two sequence

$$\{x_n\} = \left\{3 - \frac{1}{n}\right\} for \ all \ n.$$

Then $gx_n = g\left(3 + \frac{1}{n}\right)$ for all n.

Then $gx_n = g\left(3 - \frac{1}{n}\right) \longrightarrow 3$

$$\mathcal{K}x_n = \mathcal{K}\left(3 - \frac{1}{n}\right) = 6 - \left(3 - \frac{1}{n}\right) \longrightarrow 3$$
$$\hbar x_n = \hbar\left(3 + \frac{1}{n}\right) = \frac{3 + \frac{1}{n} + 3}{2} \longrightarrow 3 \text{ and } \mathscr{L}x_n = \mathscr{L}\left(3 + \frac{1}{n}\right) = \frac{2(3 + \frac{1}{n}) + 3}{2} \longrightarrow 3.$$

Therefore $lim_n gx_n = lim_n \mathcal{K}x_n = lim_n \hbar x_n = lim_n \mathscr{L}x_n = t = 3$ with $3 = \mathcal{K}3 = \mathscr{L}3$.

Also $(\mathscr{g},)\mathcal{K}$ and $(\hbar, \mathscr{L})$ are weakly compatible.

Let $\varphi: [0, \infty) \to [0, \infty)$ be a function defined by $\varphi(u) = \frac{u}{2}, for\ all\ u \in [0, \infty)$.

Then $\varphi(0) = 0$ and $0 < \varphi(u) < u, for\ all\ u \in (0, \infty)$.

Further,

$$\mathcal{G}(\mathscr{g}a, \hbar b, \hbar c, kt) < \varphi(max\{\mathcal{G}(\mathcal{K}a, \mathcal{L}b, \mathcal{L}c, t), \frac{\mathcal{G}(\mathcal{K}a, \hbar b, \hbar c, t) + \mathcal{G}(\mathcal{L}b, \hbar b, \hbar c, t) + \mathcal{G}(\hbar c, \mathcal{L}b, \mathcal{L}c, t)}{3},$$
$$\frac{\mathcal{G}(\mathcal{L}b, \hbar b, \hbar c, t) + \mathcal{G}(\mathscr{g}c, \hbar b, \hbar b, t) + \mathcal{G}(\mathscr{g}a, \hbar c, \hbar c, t)}{3}\}$$

Thus all the conditions of theorem 2.2 are satisfied and $a = 3$ is the unique common fuzzy fixed point of $\mathscr{g}, \hbar, \mathcal{K}\ and \mathscr{L}$.

Next two theorems involved with Hardy Roger's type of contractive condition for two pairs of self mappings, which extend the results contained in theorem 2.8[15], and theorem 2.2[15], theorem 3.11[6].

Theorem 2.3:

Let $(\mathbb{X}, \mathcal{G}, *)$ be a fuzzy $\mathcal{G}$- Metric space and $\mathscr{g}, \hbar, \mathcal{K}, \mathscr{L}$ be four self mappings on $\mathbb{X}$ such that

4. $(\mathscr{g}, \mathcal{K})$ and $(\hbar, \mathscr{L})$ satisfies $CLR_{(\mathcal{K}, \mathscr{L})}$ property.
5. $\mathcal{G}(\mathscr{g}a, \hbar b, \hbar c, kt) \le$
   $p\mathcal{G}(\mathscr{g}a, \mathcal{K}a, \mathcal{K}a, t) + q\mathcal{G}(\mathcal{K}a, \mathcal{L}b, \mathcal{L}b, t) + r\ \mathcal{G}(\mathcal{L}b, \hbar c, \hbar c, t) + w[\mathcal{G}(\mathscr{g}a, \mathcal{L}b, \mathcal{L}c, t) +$
   $\mathcal{G}(\mathcal{K}a, \hbar b, \hbar c, t)] \forall\ a, b, c \in \mathbb{X}$

Where $p, q, r, w \in [0,1)$ satisfying $p + q + r + 2w < 1$.

Then $(\mathscr{g}, \mathcal{K})$ and $(\hbar, \mathscr{L})$ have unique point of coincidence in $\mathbb{X}$.

Moreover, $(\mathscr{g}, \mathcal{K})$ and $(\hbar, \mathscr{L})$ are are weakly compatible, then

$\mathscr{g}, \hbar, \mathcal{K}\ and\ \mathscr{L}$ have a unique fuzzy common fixed point.

Proof:

Since $(\mathscr{g}, \mathcal{K})$ and $(\hbar, \mathscr{L})$ satiesfies $CLR_{(\mathcal{K}, \mathscr{L})}$ property, there exists two sequences

$\{x_n\}$ and $\{y_n\}$ such that $lim_n\ \mathscr{g}x_n = lim_n\ \mathscr{L}x_n = lim_n\ \hbar x_n = lim_n\ \mathscr{L}x_n = t$ with $t = \mathcal{K}a = \mathscr{L}b$ for some $t, a, \in \mathbb{X}$.

Consider,

$$\mathcal{G}(\mathscr{g}a, \hbar y_n, \hbar y_n, kt) < p\mathcal{G}(\mathscr{g}a, \mathcal{K}a, \mathcal{K}a, t) + q\mathcal{G}(\mathcal{K}a, \mathscr{L}y_n, \mathscr{L}y_n, t) + r\mathcal{G}(\mathscr{L}y_n, \hbar y_n, \hbar y_n, t) +$$
$$w[\mathcal{G}(\mathscr{g}a, \mathscr{L}y_n, \mathscr{L}y_n, t) + \mathcal{G}(\mathcal{K}a, \hbar y_n, \hbar y_n, t)]$$

On letting $n \to \infty$,

we obtain $[1 - (p + w)]\mathcal{G}(\mathscr{g}a, \ell, \ell, t) \le 0$ which gives $\mathscr{g}a = \ell = \mathcal{K}a$, since

$$p + q + r + 2w < 1.$$

Hence a is the coincidence point of $\mathscr{g}$ and $\mathcal{K}$. Similarly b is the coincidence point of $\hbar$ and $\mathscr{L}$.

Thus $l = \mathscr{g}a = \mathcal{K}a = \hbar b = \mathscr{L}b$.

Uniqueness of coincidence point:

Let l_1 and l_2 be two points of coincidence of $(\mathcal{g}, \mathcal{K})$ and $(\hbar, \mathcal{L})$.

$$l_1 = \mathcal{g}a_1 = \mathcal{K}a_1 = \hbar b_1 = \mathcal{L}b_1 \text{ and } l_2 = \mathcal{g}a_2 = \mathcal{K}b_2 = \mathcal{L}b_2.$$

Consider

$$\mathcal{G}(l_1, l_1, l_2, t) = \mathcal{G}(\mathcal{g}a_1, \hbar b_2, \hbar b_2, t) \leq$$
$$p\mathcal{G}(\mathcal{g}a_1, \mathcal{K}a_1, \mathcal{K}a_1, t) + q\mathcal{G}(\mathcal{K}a_1, \mathcal{L}b_2, \mathcal{L}b_2, t) + r\mathcal{G}(\mathcal{L}b_2, \hbar b_2, \hbar b_2, t) + w[\mathcal{G}(\mathcal{g}a_1, \mathcal{L}b_2, \hbar b_2, t) + \mathcal{G}(\mathcal{K}a_1, \hbar b_2, \hbar b_2, t)]$$

which implies $[1 - (q + 2w)]\mathcal{G}(l_1, l_1, l_2, t) \leq 0$

that is $l_1 = l_2$.

Since $(\mathcal{g}, \mathcal{K})$ and $(\hbar, \mathcal{L})$ are weakly compatible, we have

$$\mathcal{g}\mathcal{g}a = \mathcal{g}\mathcal{K}a = \mathcal{K}\mathcal{g}a = \mathcal{K}\mathcal{K}\mathcal{K}a \text{ and } \hbar\hbar b = \hbar\mathcal{L}b = \mathcal{L}\hbar b = \mathcal{L}\mathcal{L}b.$$

Now we prove that $\mathcal{g}\mathcal{g}a = \mathcal{g}a$.

Consider,

$$\mathcal{G}(\mathcal{g}\mathcal{g}a = \mathcal{g}a = \mathcal{g}a, t) = \mathcal{G}(\mathcal{g}\mathcal{g}a, \hbar b, \hbar b, t) \leq p\mathcal{G}(\mathcal{g}\mathcal{g}a, \mathcal{K}\mathcal{g}a, \mathcal{K}\mathcal{g}a, t) + q\mathcal{G}(\mathcal{K}\mathcal{g}a, \mathcal{L}b, \mathcal{L}b, t) +$$
$$r\mathcal{G}(\mathcal{L}b, \hbar b, \hbar b, t) + w[\mathcal{G}(\mathcal{g}\mathcal{g}a, \mathcal{L}b, \mathcal{L}b, t) + \mathcal{G}(\mathcal{K}\mathcal{g}a, \hbar b, \hbar b, t)]$$

Which gives $[1 - (q + 2w)]\mathcal{G}(\mathcal{g}\mathcal{g}a, \mathcal{g}a, \mathcal{g}a, t) \leq 0$.

Hence $\mathcal{g}a = \mathcal{g}\mathcal{g}a = \mathcal{K}\mathcal{g}a$.

That is $\mathcal{g}a$ is the common fuzzy fixed point of $\mathcal{g}$ and $\mathcal{K}$.

Similarly we can prove $\hbar b$ is the common fuzzy fixed point of $\hbar$ and $\mathcal{L}$.

Hence $c = \mathcal{g}c$ is the common fuzzy fixed point of $\mathcal{g}, \hbar, \mathcal{K}$ and $\mathcal{L}$.

The uniqueness of the fuzzy fixed can be proved easily.

Example 2.3:

Let $\mathbb{X} = [0,4]$ and $\mathcal{G} : \mathbb{X} \times \mathbb{X} \times \mathbb{X} \times (0, \infty) \longrightarrow [0, \infty)$ defined by $\mathcal{G}(a, b, c, t) = \max\{d(a, b, t), d(b, c, t), d(c, a, t)\}$, where d is the usual metric on $\mathbb{X}$. Then clearly $(\mathbb{X}, \mathcal{G}, *)$ is a fuzzy $\mathcal{G}$ − fuzzy metric space.

Let $\mathcal{g}, \hbar, \mathcal{K}, \mathcal{L}$ be four self maps on $\mathbb{X}$ defined by

$$\mathcal{g}a = 2, \mathcal{K}a = 4 - a, \hbar a = \frac{5a + 12}{11} \text{ and } \mathcal{L}a = \frac{a + 2}{2} \text{ for all } a \in \mathbb{X}.$$

Now $(\mathcal{g}, \mathcal{K})$ and $(\hbar, \mathcal{L})$ satisfy the $\text{CLR}_{(\mathcal{K}, \mathcal{L})}$ property.

To see this, choose two sequence

$\{x_n\} = \left\{2 + \frac{1}{n}\right\}$ and $\{y_n\} = \left\{2 + \frac{1}{n}\right\}$ for all n.

Then $gx_n = g\left(2 + \frac{1}{n}\right) \longrightarrow 2$, $\mathcal{K}x_n = \mathcal{K}\left(2 + \frac{1}{n}\right) = 4 - \left(2 + \frac{1}{n}\right) \longrightarrow 2$,

$$\hbar y_n = \hbar\left(2 - \frac{1}{n}\right) = \frac{5\left(2 - \frac{1}{n}\right) + 12}{11} \longrightarrow 2.$$

$$\mathcal{L}y_n = \mathcal{L}\left(2 - \frac{1}{n}\right) = \frac{2 - \frac{1}{n} + 2}{2} \longrightarrow 2.$$

Therefore $lim_n\, gx_n = lim_n\, \mathcal{K}x_n = lim_n\, \hbar x_n = lim_n\, \mathcal{L}x_n = t = 2$ with $2 = \mathcal{K}2 = \mathcal{L}2$.

Further, $(g, \mathcal{K})$ and $(\hbar, \mathcal{L})$ are weakly compatible and $g, \hbar, \mathcal{K}$ and $\mathcal{L}$ satisfy the contractive condition equation (2) for $p = \frac{1}{8}, q = \frac{1}{3}, r = \frac{1}{8}, w = \frac{1}{7}$.

Thus all the conditions of theorem 2.5 are satisfied and $a = 2$ is the unique common fuzzy fixed point of $g, \hbar, \mathcal{K}$ and $\mathcal{L}$.

Theorem 2.4:

Let $(\mathbb{X}, \mathcal{G}, *)$ be a fuzzy $\mathcal{G}$-Metric space and $g, \hbar, \mathcal{K}, \mathcal{L}$ be four self mappings on $\mathbb{X}$ such that

1. $(g, \mathcal{K})$ and $(\hbar, \mathcal{L})$ satisfies $\text{CLR}_{(\mathcal{K}, \mathcal{L})}$ property.
2. $\mathcal{G}(ga, \hbar b, \hbar c, kt) \le hw(a, b, c, t)$ where $h \in (0,1)$ and for all $a, b, c \in \mathbb{X}$
   $u(a, b, c) \in \left\{\mathcal{G}(ga, \mathcal{K}a, \mathcal{K}a, t), \mathcal{G}(\mathcal{K}a, \mathcal{L}b, \mathcal{L}b, t), \mathcal{G}(\mathcal{L}b, \hbar c, \hbar c, t), \frac{\mathcal{G}(ga, \mathcal{L}a, \mathcal{L}a, t) + \mathcal{G}(\mathcal{K}a, \mathcal{L}b, \mathcal{L}c, t)}{2}\right\}$
3. $(g, \mathcal{K})$ and $(\hbar, \mathcal{L})$ are are weakly compatible.

Then $g, \hbar, \mathcal{K}$ and $\mathcal{L}$ have a unique fuzzy common fixed point.

Proof:

Since $(g, \mathcal{K})$ and $(\hbar, \mathcal{L})$ satiesfies $\text{CLR}_{(\mathcal{K}, \mathcal{L})}$ property, there exists two sequences $\{x_n\}$ and $\{y_n\}$ such that $lim_n\, gx_n = lim_n\, \mathcal{K}x_n = lim_n\, \hbar y_n = lim_n\, \mathcal{L}y_n = t$ with $t = \mathcal{K}a = \mathcal{L}b$ for some $t, a, b \in \mathbb{X}$.

Consider,

$$\mathcal{G}(ga, \hbar y_n, \hbar y_n, kt) \le hw(a, \hbar y_n, \hbar y_n, t)$$

Where

$$u(a, y_n, y_n, kt)$$
$$\in \left\{\mathcal{G}(ga, \mathcal{K}a, \mathcal{K}a, t), \mathcal{G}(\mathcal{K}a, \mathcal{L}y_n, \mathcal{L}y_n, t), \mathcal{G}(\mathcal{L}y_n, \hbar y_n, \hbar y_n, t), \frac{\mathcal{G}(ga, \mathcal{L}y_n, \mathcal{L}y_n, t) + \mathcal{G}(\mathcal{K}a, \hbar y_n, \hbar y_n, t)}{2}\right\}$$

On letting $n \to \infty$,

$$\mathcal{G}(ga, w, w, t) \le hw(a, \hbar y_n, \hbar y_n, t)$$

Where $u(a, y_n, y_n, kt) \in \left\{\mathcal{G}(ga, w, w, t), \frac{\mathcal{G}(ga, w, w, t)}{2}\right\}$,

If $u(a, y_n, y_n, kt) = \mathcal{G}(ga, w, w, t)$ then $\mathcal{G}(ga, w, w, t) \le h\mathcal{G}(ga, w, w, t)$

Which also gives $ga = w = \mathcal{K}a$.

If $u(a, y_n, y_n, kt) = \frac{\mathcal{G}(ga, w, w, t)}{2}$ then $\mathcal{G}(ga, w, w, t) \le \frac{\mathcal{G}(ga, w, w, t)}{2}$ which also gives $ga = w = \mathcal{K}a$.

255

Therefore in both the cases $\mathbf{g}a = \mathcal{K}a = t$. hence a is the coincidence point of $\mathbf{g}$ and $\mathcal{K}$. similarly b is the coincidence point of $\hbar$ and $\mathscr{L}$. thus $w = \mathbf{g}a = \mathcal{K}a = \hbar b = \mathscr{L}b$.

Since $(\mathbf{g}, \mathcal{K})$ and $(\hbar, \mathscr{L})$ are are weakly compatible,

We have $\mathbf{gg}a = \mathbf{g}\mathcal{K}a = \mathcal{K}\mathbf{g}a = \mathcal{K}\mathcal{K}a$ and $\hbar\hbar b = \hbar\mathscr{L}b = \mathscr{L}\hbar b = \mathscr{L}\mathscr{L}b$.

Now prove that, $\mathbf{gg}a = \mathbf{g}a$.

Consider $\mathcal{G}(\mathbf{gg}a, \mathbf{g}a, \mathbf{g}a, t) = \mathcal{G}(\mathbf{gg}a, \hbar b, \hbar b, t) \leq hu(\mathbf{g}a, b, b, t)$,

Where $u((\mathbf{g}a, b, b, kt) \in \{\mathcal{G}(\mathbf{gg}a, \mathcal{K}\mathbf{g}a, \mathcal{K}\mathbf{g}a, t), \mathcal{G}(\mathcal{K}\mathbf{g}a, \mathscr{L}b, \mathscr{L}b, t), \mathcal{G}(\mathscr{L}b, \hbar b, \hbar b, t), \frac{\mathcal{G}(\mathbf{gg}a, \mathscr{L}b, \mathscr{L}b, t) + \mathcal{G}(\mathcal{K}\mathbf{g}a, \hbar b, \hbar b, t)}{2}$

That is $u((\mathbf{g}a, b, b, kt) = \mathcal{G}(\mathbf{gg}a, \mathbf{g}a, \mathbf{g}a, t)$.

Therefore $\mathcal{G}(\mathbf{gg}a, \mathbf{g}a, \mathbf{g}a, t) \leq h\mathcal{G}(\mathbf{gg}a, \mathbf{g}a, \mathbf{g}a, t)$ this implies $\mathbf{g}a = \mathbf{gg}a = \mathcal{K}\mathbf{g}a$.

Hence $\mathbf{g}a$ is the common fuzzy fixed point of $\mathbf{g}$ and $\mathcal{K}$. Similarly we can prove $\hbar b$ is the common fuzzy fixed point of $\hbar$ and $\mathscr{L}$.

Since $\mathbf{g}a = \hbar b, c = \mathbf{g}a$ is the common fuzzy fixed point of $\mathbf{g}, \hbar, \mathcal{K}$ and $\mathscr{L}$.

The uniqueness of the fuzzy fixed point can be proved easily.

Example 2.4:

Let $\mathbb{X} = [0,4]$ and $\mathcal{G}: \mathbb{X} \times \mathbb{X} \times \mathbb{X} \times (0, \infty) \longrightarrow [0, \infty)$ defined by $\mathcal{G}(a, b, c, t) = \max\{d(a, b, t), d(b, c, t), d(c, a, t)\}$, where d is the usual metric on $\mathbb{X}$. Then clearly $(\mathbb{X}, \mathcal{G}, *)$ is a fuzzy $\mathcal{G} -$ fuzzy metric space.

Let $\mathbf{g}, \hbar, \mathcal{K}, \mathscr{L}$ be four self maps on $\mathbb{X}$ defined by

$$\mathbf{g}a = 3, \mathcal{K}a = a, \hbar a = \frac{a+24}{9} \text{ and } \mathscr{L}a = \frac{a+3}{2} \text{ for all } a \in \mathbb{X}.$$

Now $(\mathbf{g}, \mathcal{K})$ and $(\hbar, \mathscr{L})$ satisfy the $\text{CLR}_{(\mathcal{K}, \mathscr{L})}$ property.

To see this, choose two sequence

$$\{x_n\} = \left\{3 + \frac{1}{n}\right\} \text{ and } \{y_n\} = \left\{3 - \frac{1}{n}\right\} \text{ for all n.}$$

Then $\mathbf{g}x_n = \mathbf{g}\left(3 + \frac{1}{n}\right) \longrightarrow 3$, $\mathcal{K}x_n = \mathcal{K}\left(3 + \frac{1}{n}\right) = 3 + \frac{1}{n} \longrightarrow 3$

$$\hbar y_n = \hbar\left(3 - \frac{1}{n}\right) = \frac{3 - \frac{1}{n} + 24}{9} \longrightarrow 3.$$
$$\mathscr{L}y_n = \mathscr{L}\left(3 - \frac{1}{n}\right) = \frac{3 - \frac{1}{n} + 3}{2} \longrightarrow 3.$$

Therefore $lim_n \mathbf{g}x_n = lim_n \mathcal{K}x_n = lim_n \hbar x_n = lim_n \mathscr{L}x_n = t = 3$ with $3 = \mathcal{K}3 = \mathscr{L}3$.

Further, $(\mathbf{g}, \mathcal{K})$ and $(\hbar, \mathscr{L})$ are weakly compatible and $\mathbf{g}, \hbar, \mathcal{K}$ and $\mathscr{L}$ satisfy the contractive condition equation (2) for $h = \frac{1}{2}$.

Thus all the conditions of theorem 2.4 are satisfied and $a = 3$ is the unique common fuzzy fixed point of $g, \hbar, \mathcal{K}$ and $\mathcal{L}$.

Our final end result is likewise a common fuzzy fixed factor theorem however expansive sort of contractive condition, which increase and enhance the effects theorem 3.1of [6].

Theorem 2.5:

Let $g, \hbar$ be two self maps of a fuzzy $\mathcal{G}$-Metric space $(\mathbb{X}, \mathcal{G}, *)$ satisfying $\text{CLR}_\hbar$ property and

$$\mathcal{G}(\hbar a, \hbar b, \hbar c, kt) \geq$$
$$\mathcal{A}\mathcal{G}(g a, g b, g c, t), + \mathcal{B}\mathcal{G}(g a, \hbar a, \hbar a, t) + C\ \mathcal{G}(g b, \hbar b, \hbar b, t)\ + \mathcal{D}\mathcal{G}(g c, \hbar c, \hbar c, t)\}) \ \forall\ a, b, c \in \mathbb{X}.$$

Where $\mathcal{A}, \mathcal{B}, C, \mathcal{D} > 0$. then g *and* $\hbar$ have a coincidence point. If $\mathcal{A} > 1$, then the coincidence point is unique. Moreover, if g *and* $\hbar$ are weakly compatible, then they have a unique common fuzzy fixed point.

Proof:

Since g and $\hbar$ satisfies $\text{CLR}_\hbar$ property, there exist a sequence $\{x_n\}$ in $\mathbb{X}$ such that $lim_n\ g x_n\ =\ lim_n\ \hbar x_n = \hbar a$ for some $a \in \mathbb{X}$.

Consider,

$$\mathcal{G}(g x_n, \hbar a, \hbar a, kt) \geq \mathcal{A}\mathcal{G}(g x_n, g a, g a, t) + \mathcal{B}\mathcal{G}(g x_n, \hbar x_n, \hbar x_n, t) + C\mathcal{G}(g a, \hbar a, \hbar a, t) +$$
$$\mathcal{D}\mathcal{G}(g a, \hbar a, \hbar a, t)$$

On letting $n \to \infty$,

We obtain $\left(\frac{\mathcal{A}}{2} + C + \mathcal{D}\right)\mathcal{G}(g a, \hbar a, \hbar a a, t) \leq\ 0$ which implies $g a = \hbar a$.

Thus a is coincidence point of g *and* $\hbar$.

Uniqueness of coincidence point:

Let $c = g a = \hbar a$ and $w = g b = \hbar b$ be the two points of coincidence of g *and* $\hbar$.

Then, $\mathcal{G}(c, w, w, t) = \mathcal{G}(\hbar a, \hbar b, \hbar b, t)$

$$\geq \mathcal{A}\mathcal{G}(g a, g b, g b, t), + \mathcal{B}\mathcal{G}(g a, \hbar a, \hbar a, t) + C\ \mathcal{G}(g b, \hbar b, \hbar b, t) + \mathcal{D}\mathcal{G}(g b, \hbar b, \hbar b, t)$$

That is $(\mathcal{A} - 1)\mathcal{G}(c, w, w, t) \leq 0$.

Since $\mathcal{A} > 1$, we get $c = w$.

Now $(g, \hbar)$ are weakly compatible implies $g g a = g \hbar a = \hbar g a = \hbar \hbar a$.

To prove that $g a = g g a$.

Consider $\mathcal{G}(g g a, g a, g a, t) = \mathcal{G}(\hbar g a, \hbar a, \hbar a, t)$

$$\geq \mathcal{A}\mathcal{G}(g g a, g a, g a, t), + \mathcal{B}\mathcal{G}(g g a, \hbar g a, \hbar g a, t) + C\ \mathcal{G}(g a, \hbar a, \hbar a, t) + \mathcal{D}\mathcal{G}(g a, \hbar a, \hbar a, t)$$

Which implies $g g a = g a = \hbar g a$.

Hence ga is the common fuzzy fixed point of g and h.

The forte of the fuzzy fixed point may be proved easily.

Competing Interests

Authors have declared that no competing interests exist.

References

[1]   Mustafa Z, Sims B. A New Approach to generalized metric Spaces; J. Nonlinear Convex Anal. 2006;7:289-297.

[2]   Aamri M, Moutawakil DE. Some new common fixed point theorems under strict contractive conditions, J. Math. Anal. Appl. 2002;270:181-188.

[3]   Krishnakumar R, Dhamodharan D. Fixed point theorems in normal cone metric space, International J. of Math. Sci. & Engg. Appls. (IJMSEA), ISSN 0973-9424. 2016;10(III):213-224.

[4]   Giniswamy, Maheshwari PG, Some Common Fixed Point Theorems on G-Metric Space, Gen. Math. Notes. 2014;21(2):114-124.

[5]   Mathkowski J. Fixed Point Theorems for Mappings with a Contractive Iterate at a Point, Proc. Amer. Math. Soc. 1977;62:344-348.

[6]   Kadelburg Z, Murthy PP, Radenovic S. Common fixed points for expansive mappings in cone metric spaces, Int. J. Math. Anal. 2011;5(27):1309-1319.

[7]   Mehta JG, Joshi ML. On complete cone metric space and fixed point theorems, J. Sci. Res. 2011;3(2):303-309.

[8]   Senthil Kumar P, Thiruveni P. Fuzzy Fixed Point Theorems in Normal Cone Metric Space, Asian Research journal of Mathematics. 2023;19(10):57-66.

[9]   Senthil Kumar P, Thiruveni P. Estimation of fuzzy metric spaces from metric spaces, Ratio Mathematica. 2021;41:64-70.

[10]   Senthil kumar P, Thiruveni P, Dhamodharan D. Some results on common fixed point in fuzzy 2-normed linear space, Advances And Applications In Mathematical Sciences. 2022;21(11):6425-6435.

[11]   Senthil kumar P, Thiruveni P, Rostam K Saeed. Fuzzy Fixed Point Theorems for γ – Weak Contractive Mappings in 3 Metric Spaces, Journal of Algebraic Statistics. 2022;13(2):723-729SN: 1309-3452.

[12]   Mohamed Ali A, RanjithKanna G, Intuitionistic Fuzzy Cone Metric Spaces and Fixed Point Theorems, International Journal of Mathematics And its Applications. 2017;5(1–A):25–36.

[13]   Senthil Kumar. P and Thiruveni. P, Novel Way of C^*- Algebra Valued 2- metric Space, Proceedings of the International Conference on Recent Innovations in Applications of Mathematics, 2023, 194 – 201.

[14]   Maheshwari PG. Some Common Fixed Point Theorems on G-Metric Space. General Mathematics Notes. 2014 Apr 1;21(2).

[15]   Abbas M, Rhoades BE, Nazir T. Common fixed points for four maps in cone metric spaces, Appl. Math. Comput. 2010;216(1):80-86.

_____