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A Discourse on Completely Regular Space

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Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

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ABSTRACT

This study is an investigation of some of the relationships which exist between various generalizations of completely regular spaces. The primary aim of the study is to look at the separation axioms and delve more into one of the claims about completely regular space; "Every completely regular space is a regular space as well".

Keywords: Topological space; neighbourhood; open sets; closed sets; continuous map; Kolmogorov space; Fréchet space; Hausdorff space; regular space; normal space; completely regular space.

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1. INTRODUCTION

Topology is popularly considered to have begun with Leonhard Euler's (1707 - 1783) solution to the famous Königsberg bridges problem but the term "topology" first appeared in the title of Listing's paper, "Vorstudien zur Topologie" in 1847 [1]. Topology is one of the branches of mathematics which is growing rapidly fast. Topology is used in modeling and understanding

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real life structures and phenomena. Topology grew out of geometry. It is the study of the properties of shapes, deformations applied to the shapes and relationship between them. The study of topology is very useful and applicable in many areas especially, cosmology.

Separation axioms that explain the features of a topological space are extremely general and weak [2]. The separation axioms involve “separating” certain kinds of sets from one another by disjoint open sets [3].

In this paper, we will briefly discuss the separation axioms with more emphasis on completely regular space and some claims about it. The following preliminaries serve this purpose.

2. PRELIMINARIES

Definition 2.1

Let X be a non – empty set. A topology τ on X is a collection of subsets of X, each called an open set, such that

(i) $\varnothing$ and X are open sets;
(ii) The intersection of finitely many open sets is an open set;
(iii) The union of any collection of open sets is an open set.

The pair (X, τ) is called a topological space [1].

Definition 2.2

A topological space (X, τ) is said to be T_o or Kolmogorov if, whenever $x \neq y$, there ***either*** exists an open set U with $x \in U$, $y \notin U$ or there exists an open set V with $y \in V$, $x \notin V$ [4].

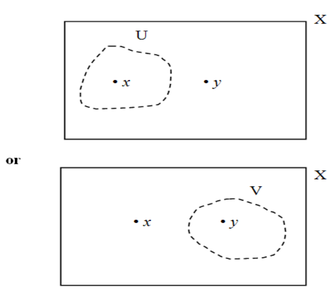

Fig. 1. T_o or Kolmogorov space

Definition 2.3

A topological space (X, τ) is said to be T_1 or Fréchet space if given $a, b \in X$ and $a \neq b$, there exists open sets U_a, U_b

$\in \tau$ containing *a, b* respectively, such that $b \notin U_a$ and $a \notin U_b$ [4].

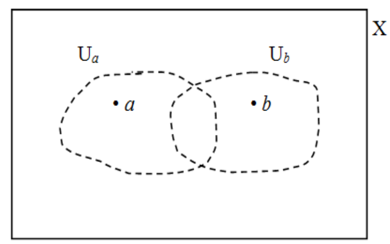

Fig. 2. T_1 or Fréchet space

Definition 2.4

A topological space (X, τ) is said to be T_2 or Hausdorff space if given $a, b \in X$ and $a \neq b$, there exists ***disjoint*** open sets $U, V \in X$ such that $a \in U$ and $b \in V$ [4].

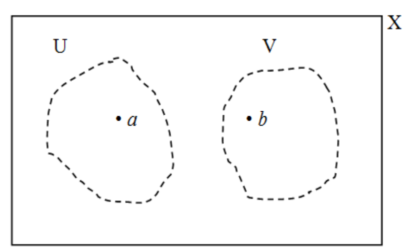

Fig. 3. T_2 or Hausdorff space

Definition 2.5

A T_1 – space is called a T_3 – space or regular if for any point *x* and closed set F not containing *x*, there are ***disjoint*** open sets U and V with $x \in U$ and $F \subset V$ [2].

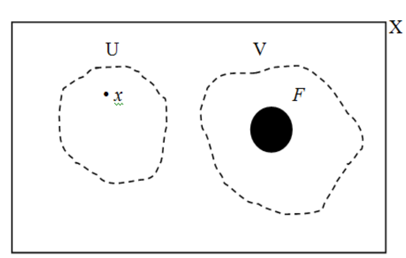

Fig. 4. T_3 or Regular space

Definition 2.6

A topological space (X, τ) is said to be T_4 or normal space if *A* and *B* are ***disjoint*** closed sets in X, there exists ***disjoint*** open sets $U_A, U_B \in X$ containing *A* and *B* respectively

(ie. $U_A \cap U_B = \varnothing$) [3].

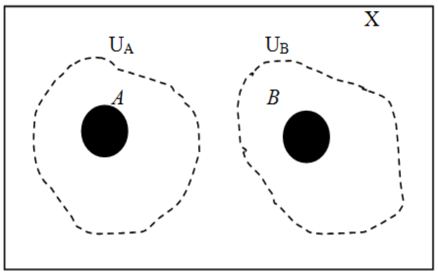

Fig. 5. T_4 or Normal space

3. MAIN RESULTS

Definition 3.1

A topological space (X, τ) is said to be completely regular if given closed set C of X and a point $x \in X$ such that $x \notin C$, then there exists a continuous map $f : X \longrightarrow [0,1]$ such that $f(x) = 0$ and $f(C) = \{1\}$. Completely regular space is also known as $T_{3\frac{1}{2}}$ or Tychonoff space [5].

Theorem

Every completely regular space is a regular space as well [5].

Proof (Suggested)

Let (X, τ) be a completely regular space, and let $x \in X$ and C be a closed subset of X such that $x \notin C$, then $f(x) = 0$ and $f(C) = \{1\}$. From this definition, it implies $f(x) \cap f(C) = \varnothing$.

Choose $U_{f(x)}$ and $V_{f(C)}$ as open sets for *f(x)* and *f(C)* respectively such that $U_{f(x)} \cap V_{f(C)} = \varnothing$. Due to continuity of the function, $f^{-1}(U_{f(x)})$ and $f^{-1}(V_{f(C)})$ are open in the topological space since $U_{f(x)}$ and $V_{f(C)}$ are open sets. Then $f^{-1}(U_{f(x)})$ and $f^{-1}(V_{f(C)})$ are ***disjoint*** open sets of *x* and C respectively

(ie. $f^{-1}(U_{f(x)}) \cap f^{-1}(V_{f(C)}) = \varnothing$). Hence (X, τ) satisfies **Definition 2.5,** then (X, τ) is regular.

4. CONCLUSION

From the proof above it can be concluded that any topological space which is completely regular is a regular space as well.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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