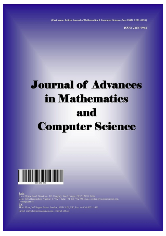

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Existence of the Rotational Subsonic Stationary Solution for a Two-Dimensional Bipolar Euler-Poisson Equation

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Authors' contributions

This work was carried out in collaboration between Bth authors. Author FL designed the study, performed the statistical analysis, wrote the protocol, and wrote the first draft of the manuscript. Authors YL managed the analyses of the study and managed the literature searches. Both authors read and approved the final manuscript.

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Abstract

In this paper, we study a two-dimensional bipolar Euler-Poisson equation (hydrodynamic model), which arises in mathematical modeling for semiconductors and plasmas. We are interested in the existence of the rotational subsonic stationary solution. Under the proper boundary conditions, we show the existence of rotational subsonic stationary solutions for the two-dimensional bipolar Euler-Poisson equation. This result is the first result about the rotational subsonic stationary solution for the multi-dimensional bipolar isentropic Euler-Poisson equation. The proof is completed by delicate energy estimate and fixed point principle.

Keywords: Euler-Poisson equation; rotational stationary solution; energy estimate.

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1 Introduction

In this paper, we are concerned with the transient bipolar Euler-Poisson system, which is written as (see [1, 2, 3])

$$\begin{cases} \partial_t n_1 + \mathrm{div}(n_1 u_1) = 0, \\ \varepsilon \partial_t (n_1 u_1) + \varepsilon \mathrm{div}(n_1 u_1 \otimes u_1) + \nabla P(n_1) = n_1 \nabla \phi - \frac{\varepsilon n_1 u_1}{\tau}, \\ \partial_t n_2 + \mathrm{div}(n_2 u_2) = 0 \\ \mu \partial_t (n_2 u_2) + \mu \mathrm{div}(n_2 u_2 \otimes u_2) + \nabla P(n_2) = -n_2 \nabla \phi - \frac{\mu n_2 u_2}{\tau}, \\ \lambda^2 \Delta \phi = n_1 - n_2. \end{cases} \tag{1.1}$$

The unknown functions $n_i,\ u_i (i = 1, 2)$, and ϕ are the charge densities, velocities, and electrostatic potential. The constant coefficients ε and μ denote the scaled electron mass and the hole mass respectively, and $\lambda > 0$ stands for the Debye-length. The functions $P(n_1)$ and $P(n_2)$ are the pressure-density relations which satisfy that $n_1^2 P'(n_1), n_2^2 P'(n_2)$ are strictly monotonically increasing from $[0, \infty)$ onto $[0, \infty)$. A commonly used hypothesis is $P(n_i) = kn_i^\gamma (i = 1, 2, \gamma \geq 1, k > 0)$. The current densities J_1, J_2 are given by

$$J_1 = -n_1 u_1,\ J_2 = -n_2 u_2.$$

$\tau (> 0)$ represents the velocity relaxation time and is modeled as function of J_1, J_2, n_1, n_2 and x:

$$\tau = \tau(J_1, J_2, n_1, n_2, x).$$

One application of the hydrodynamic models (Euler-Poisson equations) is to describe the transport of charged fluid particles such as electrons and holes in semiconductor devices or positively and negatively charged ions in a plasma. These models can be derived from kinetic models, and take an important place in the fields of applied physics and computational mathematics. According to the different ansatz for the phase space densities, introduced to prescribe the dependence on the velocity, we recover different limit models and, in particular, the drift-diffusion equations and the hydrodynamic (Euler-Poisson) systems. More details on the bipolar Euler-Poisson equations can be founded in, e.g., [2, 3] and some reference therein.

Recently, there are many studies on the subsonic, supersonic and transonic stationary solution of the unipolar Euler-Poisson equations. More precisely, Degond and Markowich [4] and Fang and Ito [5] discussed the well-posedness of the subsonic stationary solutions for the one-dimensional unipolar isentropic Euler-Poisson equations, respectively. Peng and Violet [6] given an example for the supersonic stationary solutions for the one-dimensional isentropic unipolar Euler-Poisson equations. Ascher, etc. [7] and Rosin [8] investigated the transonic flow for the unipolar isentropic Euler-Poisson equations with a linear pressure function, and the special boundary conditions and the special doping profile, by phase plane analysis. A transonic solution which may contain transonic shocks was constructed by Gamba [9] by using a vanishing viscosity limit method. Luo and Xin [10] given a thorough study on the existence, structure and location of the transonic stationary solution for the unipolar Euler-Poisson equations. Gamba and Morawetz [11] studied a viscous approximation of transonic solution in the two dimensional semiconductor equations. Degond and Markowich [12] and Yeh [13] established the irrotational subsonic stationary solutions for a multi-dimensional unipolar isentropic Euler-Poisson equations, respectively. Markowich [14] showed the existence of rotational subsonic solutions for the two-dimensional steady-state Euler-Poisson equations. Amater and Beccar Varela [15] showed the existence of the subsonic stationary solution to a one-dimensional non-isentropic unipolar Euler-Poisson equations. Markowich and Pietra [16] discussed the transonic flow for the unipolar non-isentropic Euler-Poisson equations with a linear pressure function, and the special boundary conditions and the special doping profile, by phase plane analysis. Li and Zhang [17] studied the irrotational subsonic stationary solutions for a multi-dimensional non-isentropic Euler-Poisson equations. However, the study of the bipolar Euler-Poisson equation is far from being mature. Tsuge [18] and Zhou and Li [19] discussed the unique existence of the subsonic stationary solution for the one-dimensional bipolar Euler-Poisson equation, respectively. Cordier, et al. studied the traveling wave solutions of the bipolar isentropic and non-isentropic Euler-Poisson equations in [20, 1], where the stationary traveling wave solution may contain the transonic shock. Li [21] showed the unique existence of the subsonic irrotational stationary solution for the multi-dimensional bipolar Euler-Poisson equation. Motivated by [21, 14], we will show the existence of rotational subsonic stationary solutions for the two-dimensional bipolar Euler-Poisson equation in this paper.

First, when $n_{1t} = u_{1t} = n_{2t} = u_{2t} = 0$, the corresponding stationary bipolar Euler-Poisson equations of the system (1.1) can be written as:

$$\begin{cases} \operatorname{div}(n_1 u_1) = 0, \\ \varepsilon(u_1 \cdot \nabla)u_1 + \nabla(h(n_1) - \phi) = -\frac{u_1}{\tau}, \\ \operatorname{div}(n_2 u_2) = 0, \\ \mu(u_2 \cdot \nabla)u_2 + \nabla(h(n_2) + \phi) = -\frac{u_2}{\tau}, \\ \Delta\phi = n_1 - n_2 \end{cases} \tag{1.2}$$

for $x \in \Omega$. Here Ω be an open and bounded domain of $\mathbb{R}^2$, and h is the enthalpy function:

$$h'(s) = \frac{p'(s)}{s}, \ s > 0 \text{ and } h(1) = 0.$$

We also prescribe the following boundary conditions:

$$n_1 = n_{1D}, \ n_2 = n_{2D}, \ J_1 \cdot \nu = J_{1D}, \ J_2 \cdot \nu = J_{2D}, \ u_1 = -\frac{J_{1D}}{n_{1D}}, \ u_2 = -\frac{J_{2D}}{n_{2D}} \ \text{ on } \partial\Omega, \tag{1.3}$$

where ν denotes the outward unit normal of $\partial\Omega$. Clearly, in accordance with $(1.2)_1$ and $(1.2)_3$ we also require

$$\int_{\partial\Omega} J_{iD} ds = 0 \quad (i = 1, 2). \tag{1.4}$$

Before stating our results, we first give some assumptions:

(H1) Ω is a bounded and convex of $\mathbb{R}^2$ with $\partial\Omega \in C^{2,\delta}$, $\delta \in (0, 1)$,
(H2) $p \in C^3(\mathbb{R}^+)$, and $p'(n_i) > 0$, $\forall\ n_i > 0$, $i = 1, 2$,
(H3) $\tau = \tau(J_1, J_2, n_1, n_2, x), \tau \in C^2(\mathbb{R}^2 \times \mathbb{R}^2 \times \mathbb{R}^+ \times \mathbb{R}^+ \times \overline{\Omega})$, $\exists \tau_1, \tau_2, \tau_3, \tau_4 > 0$, s.t. $\tau_1 \le \tau \le \tau_2, |\nabla_{J_1}\tau| + |\nabla_{J_2}\tau| < \tau_3, |\frac{\partial\tau}{\partial n_1}| + |\frac{\partial\tau}{\partial n_2}| + |\nabla_x \tau| < \tau_4$, $\forall (J_1, J_2, n_1, n_2, x) \in \mathbb{R}^2 \times \mathbb{R}^2 \times \mathbb{R}^+ \times \mathbb{R}^+ \times \overline{\Omega}$,
(H4) $n_{1D}, n_{2D} \in W^{2,q}(\Omega)$ for some integer $q > \frac{2}{1-\delta}$ and $\exists \underline{N}, \overline{N} > 0$, s.t. $0 < \underline{N} < n_{1D}, n_{2D} < \overline{N}, \forall x \in \partial\Omega$,
(H5) $u_{iD} = -\frac{J_{iD}}{n_{iD}} \in W^{2,2q}(\Omega)$, $\int_{\partial\Omega} J_{iD} ds = 0 (i = 1, 2)$ and $\|(J_{1D}, J_{2D})\|_{W^{2,2q}}$ is small enough.

The assumptions guarantee a fully subsonic flow and allow us to control the vorticity of u_1 and u_2. Now we state the main result of this paper by the following theorem.

Theorem 1.1. *Let the assumptions (H1)-(H5) hold. Then the problem (1.2)-(1.3) has a solution* $(n_1, u_1, n_2, u_2, \phi) \in C^{1,\delta}(\overline{\Omega}) \times W^{1,2q}(\overline{\Omega}) \times C^{1,\delta}(\overline{\Omega}) \times W^{1,2q}(\overline{\Omega}) \times C^{1,\delta}(\overline{\Omega})$.

Remark 1.2. Here we only obtain the existence of the rotational subsonic stationary solution, but the uniqueness is open. Moreover, it is an important and interesting to study the supersonic and transonic stationary solution for the bipolar Euler-Poisson equations. Finally, it would be interesting to investigate the stability of the rotational subsonic stationary solutions obtained in the paper as in [22, 23, 24, 25]. These are what our effort should aim at in the forthcoming future.

The rest of this paper is organized as follows. In the next section, we make some preliminaries. That is, we reduce the problem (1.2)-(1.3) to a series of the boundary value problem of the second-order elliptic equations. We show the existence of the subsonic rotational stationary solutions of the problem (1.2) with the boundary value (1.3).

2 Preliminaries

In this section, we make some preliminaries. First, motivated by [14], let us introduce the following regularized system of (1.2) which facilitate the analysis to a large extent:

$$\begin{cases} -\alpha^2\Delta u_1^\alpha + \alpha^2\nabla(\mathrm{div} u_1^\alpha) + \varepsilon(u_1^\alpha\cdot\nabla)u_1^\alpha + \nabla(h(n_1^\alpha) - \phi^\alpha) = -\frac{u_1^\alpha}{\tau^\alpha}, \\ \mathrm{div}(n_1^\alpha u_1^\alpha) = 0, \\ -\alpha^2\Delta u_2^\alpha + \alpha^2\nabla(\mathrm{div} u_2^\alpha) + \mu(u_2^\alpha\cdot\nabla)u_2^\alpha + \nabla(h(n_2^\alpha) + \phi^\alpha) = -\frac{u_2^\alpha}{\tau^\alpha}, \\ \mathrm{div}(n_2^\alpha u_2^\alpha) = 0, \\ \Delta\phi^\alpha = n_1^\alpha - n_2^\alpha, \end{cases} \tag{2.1}$$

subject to the boundary conditions:

$$n_1^\alpha = n_{1D},\ n_2^\alpha = n_{2D},\ u_1^\alpha\cdot\nu = u_{1D},\ u_2^\alpha\cdot\nu = u_{2D},\ \mathrm{curl} u_1^\alpha = 0,\ \mathrm{curl} u_2^\alpha = 0 \ \text{ on } \partial\Omega, \tag{2.2}$$

and we set $\tau^\alpha := \tau(-n_1^\alpha u_1^\alpha, -n_2^\alpha u_2^\alpha, n_1^\alpha, n_2^\alpha, x)$.

Obviously, the divergence and curl of the regularization term $-\alpha^2\Delta u_i^\alpha + \alpha^2\nabla(\mathrm{div} u_i^\alpha)$ equal to zero and $-\alpha^2\Delta u_i^\alpha$ respectively.

Next, denoting $w_i^\alpha = \mathrm{curl} u_i^\alpha$ and $u_{i\perp}^\alpha := (-u_{i2}^\alpha, u_{i1}^\alpha)$ for $i = 1, 2$, and taking the curl of $(2.1)_1$ and $(2.1)_3$, we have

$$\begin{cases} -\alpha^2\Delta w_1^\alpha + \varepsilon u_1^\alpha\cdot\nabla w_1^\alpha + (\varepsilon \mathrm{div} u_1^\alpha + \frac{1}{\tau^\alpha})w_1^\alpha = -\nabla(\frac{1}{\tau^\alpha})\cdot u_{1\perp}^\alpha, \\ -\alpha^2\Delta w_2^\alpha + \mu u_2^\alpha\cdot\nabla w_2^\alpha + (\mu \mathrm{div} u_2^\alpha + \frac{1}{\tau^\alpha})w_2^\alpha = -\nabla(\frac{1}{\tau^\alpha})\cdot u_{2\perp}^\alpha, \\ w_1^\alpha = 0,\ w_2^\alpha = 0 \ \text{ on } \partial\Omega. \end{cases} \tag{2.3}$$

On the other hand, taking the divergence of $(2.1)_1$ and $(2.1)_3$, then using $(2.1)_2$, $(2.1)_4$ and $(2.1)_5$ gives

$$\begin{cases} \Delta h(n_1^\alpha) - \frac{\varepsilon}{n_1^\alpha}\sum_{i,j=1}^2 u_{1i}^\alpha u_{1j}^\alpha n_{1x_ix_j}^\alpha + \frac{\varepsilon}{(n_1^\alpha)^2}(\nabla n_1^\alpha\cdot u_1^\alpha)^2 - \frac{\varepsilon}{n_1^\alpha}\sum_{i,j=1}^2 u_{1i}^\alpha (u_{1j}^\alpha)_{x_i}(n_1^\alpha)_{x_j} \\ \qquad -\frac{1}{\tau^\alpha}\frac{\nabla n_1^\alpha}{n_1^\alpha}\cdot u_1^\alpha - (n_1^\alpha - n_2^\alpha) = -\varepsilon\sum_{i,j=1}^2 (u_{1i}^\alpha)_{x_j}(u_{1j}^\alpha)_{x_i} - \nabla(\frac{1}{\tau^\alpha})\cdot u_1^\alpha, \\ \Delta h(n_2^\alpha) - \frac{\mu}{n_2^\alpha}\sum_{i,j=1}^2 u_{2i}^\alpha u_{2j}^\alpha n_{2x_ix_j}^\alpha + \frac{\mu}{(n_2^\alpha)^2}(\nabla n_2^\alpha\cdot u_2^\alpha)^2 - \frac{\mu}{n_2^\alpha}\sum_{i,j=1}^2 u_{2i}^\alpha (u_{2j}^\alpha)_{x_i}(n_2^\alpha)_{x_j} \\ \qquad -\frac{1}{\tau^\alpha}\frac{\nabla n_2^\alpha}{n_2^\alpha}\cdot u_2^\alpha + (n_1^\alpha - n_2^\alpha) = -\mu\sum_{i,j=1}^2 (u_{2i}^\alpha)_{x_j}(u_{2j}^\alpha)_{x_i} - \nabla(\frac{1}{\tau^\alpha})\cdot u_2^\alpha, \\ n_1^\alpha = n_{1D},\ n_2^\alpha = n_{2D} \ \text{ on } \partial\Omega. \end{cases} \tag{2.4}$$

Then we can regard (2.3) as elliptic problems for w_1^α, w_2^α and (2.4) as elliptic problems for n_1^α, n_2^α.

Finally, in order to control the curl-free part of u_1^α, u_2^α, we split u_1^α, u_2^α in the way as usual

$$u_1^\alpha = -\nabla\psi_1^\alpha + \sigma_1^\alpha,\ u_2^\alpha = -\nabla\psi_2^\alpha + \sigma_2^\alpha, \tag{2.5}$$

where $\text{div}\sigma_i^\alpha = 0$ in Ω, $\sigma_i^\alpha \cdot \nu = 0$ and $\nabla\psi_i^\alpha \cdot \nu = -u_{iD}$ on $\partial\Omega,\ i = 1, 2$. Let us define $(-\mu_{ix_2}, \mu_{ix_1})^T$ for $i = 1, 2$, then we have

$$\begin{cases} \Delta\mu_i = w_i^\alpha,\ x \in \Omega, \\ \mu_i|_{\partial\Omega} = 0, \end{cases} \tag{2.6}$$

and

$$\begin{cases} \text{div}(n_1^\alpha \nabla\psi_1^\alpha) = \text{div}(n_1^\alpha \sigma_1^\alpha), \quad \nabla\psi_1^\alpha \cdot r|_{\partial\Omega} = -u_{1D}, \\ \text{div}(n_2^\alpha \nabla\psi_2^\alpha) = \text{div}(n_2^\alpha \sigma_2^\alpha), \quad \nabla\psi_2^\alpha \cdot r|_{\partial\Omega} = -u_{2D}. \end{cases} \tag{2.7}$$

In order to obtain a unique solution ψ_i^α, we require

$$\int_\Omega \psi_i^\alpha dx = 0,\ i = 1, 2. \tag{2.8}$$

It is easy to see that the existence of system (2.1)-(2.2) and (2.3)-(2.8) is equivalent.

3 Existence of the Subsonic Rotational Stationary Solutions

In this section, we give the proof of the subsonic rotational stationary solution of (1.2)-(1.3). Firstly, we use the Schauder fixed point theorem to establish the solutions of (2.3)-(2.8). For convenience, we will skip the superscript α. For this aim, we define the following closed and convex sets:

$$A = \{(m_1, m_2) \in C^{1,\delta}(\overline{\Omega}) \times C^{1,\delta}(\overline{\Omega}) : \underline{N} \le m_1, m_2 \le \overline{N}, \|(m_1, m_2)\|_{C^{1,\delta}} \le N\},$$
$$B = \{(v_1, v_2) \in W^{1,2q}(\Omega) \times W^{1,2q}(\Omega) : \|(v_1, v_2)\|_{W^{1,2q}(\Omega)} \le \gamma, \|(\text{div}v_1, \text{div}v_2)\|_{L^\infty(\Omega)} \le \gamma\},$$

here $N > 0$ and $\gamma > 0$ are constant to be defined later. Choosing $(m_1, m_2, v_1, v_2) \in A \times B$, where $v_1 = (v_{11}, v_{12}), v_2 = (v_{21}, v_{22})$, we can construct the fixed operator $T : (m_1, m_2, v_1, v_2) \to (n_1, n_2, u_1, u_2)$ as follows: First, solve

$$\begin{cases} \Delta\xi_1 - \frac{\varepsilon}{p'(m_1)} \sum_{i,j=1}^2 v_{1i}v_{1j}\xi_{1x_ix_j} - \varepsilon\frac{g''(h(m_1))}{m_1 g'(h(m_1))} \sum_{i,j=1}^2 v_{1i}v_{1j}(m_1)_{x_i}(\xi_1)_{x_j} \\ \quad +\varepsilon\frac{g'(h(m_1))}{m_1^2}(\nabla m_1 \cdot v_1)v_1 \cdot \nabla\xi_1 - \frac{1}{\tau_0 p'(m_1)} v_1 \cdot \nabla\xi_1 - (g(\xi_1) - g(\xi_2)) \\ = -\varepsilon \sum_{i,j=1}^2 (v_{1i})_{x_j}(v_{1j})_{x_i} + \frac{\varepsilon}{m_1} \sum_{i,j=1}^2 v_{1i}(v_{1j})_{x_i}(m_1)_{x_j} - \nabla(\frac{1}{\tau_0}) \cdot u_1, \\ \xi_1 = h(n_{1D}) \quad \text{on } \partial\Omega, \end{cases} \tag{3.1}$$

and

$$\begin{cases} \Delta\xi_2 - \frac{\mu}{p'(m_2)} \sum_{i,j=1}^2 v_{2i}v_{2j}\xi_{2x_ix_j} - \mu\frac{g''(h(m_2))}{m_2 g'(h(m_2))} \sum_{i,j=1}^2 v_{2i}v_{2j}(m_2)_{x_i}(\xi_2)_{x_j} \\ \quad +\mu\frac{g'(h(m_2))}{m_2^2}(\nabla m_2 \cdot v_2)v_2 \cdot \nabla\xi_2 - \frac{1}{\tau_0 p'(m_2)} v_2 \cdot \nabla\xi_2 + (g(\xi_1) - g(\xi_2)) \\ = -\mu \sum_{i,j=1}^2 (v_{2i})_{x_j}(v_{2j})_{x_i} + \frac{\mu}{m_2} \sum_{i,j=1}^2 v_{2i}(v_{2j})_{x_i}(m_1)_{x_j} - \nabla(\frac{1}{\tau_0}) \cdot u_2, \\ xi_2 = h(n_{2D}) \quad \text{on } \partial\Omega, \end{cases} \tag{3.2}$$

where $g = h^{-1}$ is the inverse function of h and $\tau_0 := \tau(-m_1v_1, -m_2v_2, m_1, m_2, x)$. Then compute n_i from $n_i = g(\xi_i),\ i = 1, 2$.

Next, solve

$$\begin{cases} -\alpha^2\Delta w_1 + \varepsilon v_1 \cdot \nabla w_1 + (\varepsilon\text{div}v_1 + \frac{1}{\tau_0})w_1 = -\nabla(\frac{1}{\tau_0}) \cdot v_{1\perp}, \\ -\alpha^2\Delta w_2 + \mu v_2 \cdot \nabla w_2 + (\mu\text{div}v_2 + \frac{1}{\tau_0})w_2 = -\nabla(\frac{1}{\tau_0}) \cdot v_{2\perp}, \\ w_1 = 0,\ w_2 = 0 \quad \text{on } \partial\Omega, \end{cases} \tag{3.3}$$

for w_1, w_2.

Finally, solve

$$\Delta\mu_i = w_i,\ \mu_i|_{\partial\Omega} = 0,\ i = 1, 2, \tag{3.4}$$

and set $\sigma_i = \begin{pmatrix} -\mu_{ix_2} \\ \mu_{ix_1} \end{pmatrix}$, $i = 1, 2$, and compute ψ_i from

$$\mathrm{div}(n_i \nabla\psi_i) = \nabla n_i \cdot \sigma_i,\ \nabla\psi_i \cdot r|_{\partial\Omega} = -u_{iD},\ \int_\Omega \psi_i dx = 0, \tag{3.5}$$

and set $u_i = -\nabla\psi_i + \sigma_i$.

Therefor, in the following, we only need to show that $T : A \times B \to A \times B$ is a continuous compact operator when the parameters N, γ are chosen appropriately.

At first, about (3.1) and (3.2), we have

Lemma 3.1. Let the assumptions (H1)-(H5) hold, then there exist $\varepsilon_1, \mu_1, \gamma_1 > 0$ such that the problem (3.1) and (3.2) has a unique solution $(n_1, n_2) \in A$, for all $\varepsilon \in [0, \varepsilon_1]$, $\mu \in [0, \mu_1]$ and $\gamma \in [0, \gamma_1]$.

Proof. The problem (3.1) and (3.2) is elliptic equations if and only if

$$|v_1| < \sqrt{p'(m_1)/\varepsilon}, |v_2| < \sqrt{p'(m_2)/\mu},$$

this is certainly holds for all $\varepsilon \in [0, \varepsilon_1]$, $\mu \in [0, \mu_1]$ if $\gamma \le \frac{C_1}{2} \min_{\underline{N} \le m \le \overline{N}} \sqrt{p'(m)/\epsilon}$, where C_1 is the bound of the imbedding $W^{1,2q}(\Omega) \to L^\infty(\Omega)$ and $\epsilon = \max(\varepsilon_1, \mu_1)$. Then, from the existence theories of the second-order elliptic equations in [26, ?], we can obtain a unique solution $(\xi_1, \xi_2) \in W^{2,q}(\Omega) \times W^{2,q}(\Omega)$. Set $(\xi_1 - h(\overline{N}))^+ = \max(\xi_1 - h(\overline{N}), 0)$ and $(\xi_2 - h(\overline{N}))^+ = \max(\xi_2 - h(\overline{N}), 0)$. Multiplying $(3.1)_1$ by $(\xi_1 - h(\overline{N}))^+$, $(3.2)_1$ by $(\xi_2 - h(\overline{N}))^+$ respectively, and integrating them over Ω, we have

$$\begin{aligned}
&\int_\Omega |\nabla(\xi_1 - h(\overline{N}))^+|^2 dx - \int_\Omega \frac{\varepsilon}{p'(m_1)} \sum_{i,j=1}^{2} v_{1i} v_{1j} (\xi_1 - h(\overline{N}))_{x_i} (\xi_1 - h(\overline{N}))^+_{x_j} dx \\
&- \int_\Omega \sum_{i,j=1}^{2} (\frac{\varepsilon}{p'(m_1)} v_{1i} v_{1j})_{x_j} (\xi_1 - h(\overline{N}))_{x_i} (\xi_1 - h(\overline{N}))^+ dx \\
&+ \int_\Omega \frac{\varepsilon g''(h(m_1))}{m_1 g'(h(m_1))} \sum_{i,j=1}^{2} v_{1i} v_{1j} (m_1) x_i (\xi_1 - h(\overline{N}))_{x_j} (\xi_1 - h(\overline{N}))^+ dx \\
&- \int_\Omega (\varepsilon \frac{g'(h(m_1))}{m_1^2} (\nabla m_1 \cdot \nabla v_1) v_1 - \frac{v_1}{\tau_0 p'(m_1)}) \cdot \nabla(\xi_1 - h(\overline{N}))(\xi_1 - h(\overline{N}))^+ dx \\
&+ \int_\Omega (g(\xi_1) - g(\xi_2))(\xi_1 - h(\overline{N}))^+ dx \\
= \ &\int_\Omega (\varepsilon \sum_{i,j=1}^{2} (v_{1i})_{x_j} (v_{1j})_{x_i} - \frac{\varepsilon}{m_1} \sum_{i,j=1}^{2} v_{1i} (v_{1j})_{x_i} (m_1)_{x_j} + \nabla(\frac{1}{\tau_0}) \cdot v_1)(\xi_1 - h(\overline{N}))^+ dx,
\end{aligned}$$

and

$$\begin{aligned}
&\int_\Omega |\nabla(\xi_2 - h(\overline{N}))^+|^2 dx - \int_\Omega \frac{\mu}{p'(m_2)} \sum_{i,j=1}^{2} v_{2i}v_{2j}(\xi_2 - h(\overline{N}))_{x_i}(\xi_2 - h(\overline{N}))^+_{x_j} dx \\
&- \int_\Omega \sum_{i,j=1}^{2} (\frac{\mu}{p'(m_2)} v_{2i}v_{2j})_{x_j}(\xi_2 - h(\overline{N}))_{x_i}(\xi_2 - h(\overline{N}))^+ dx \\
&+ \int_\Omega \frac{\mu g''(h(m_2))}{m_2 g'(h(m_2))} \sum_{i,j=1}^{2} v_{2i}v_{2j}(m_2)x_i(\xi_2 - h(\overline{N}))_{x_j}(\xi_2 - h(\overline{N}))^+ dx \\
&- \int_\Omega (\mu \frac{g'(h(m_2))}{m_2^2}(\nabla m_2 \cdot \nabla v_2)v_2 - \frac{v_2}{\tau_0 p'(m_2)}) \cdot \nabla(\xi_2 - h(\overline{N}))(\xi_2 - h(\overline{N}))^+ dx \\
&- \int_\Omega (g(\xi_1) - g(\xi_2))(\xi_2 - h(\overline{N}))^+ dx \\
= \;& \int_\Omega (\mu \sum_{i,j=1}^{2} (v_{2i})_{x_j}(v_{2j})_{x_i} - \frac{\mu}{m_2} \sum_{i,j=1}^{2} v_{2i}(v_{2j})_{x_i}(m_2)_{x_j} + \nabla(\frac{1}{\tau_0}) \cdot v_2)(\xi_2 - h(\overline{N}))^+ dx.
\end{aligned}$$

Noting

$$\|\nabla(\frac{1}{\tau_0})\|_{L^q(\Omega)} \leq \frac{1}{\tau_1^2}\|\tau_0\|_{L^q(\Omega)} \leq \frac{C}{\tau_1^2}(\tau_4(1+N) + \gamma\tau_3(N+1)),$$

and applying Cauchy-Schwarz inequality to the above equalities, we can obtain

$$\begin{aligned}
&\int_\Omega |\nabla(\xi_1 - h(\overline{N}))^+|^2 dx + \int_\Omega (g(\xi_1) - g(\xi_2))(\xi_1 - h(\overline{N}))^+ dx \\
\leq \;& C_1\gamma(\int_\Omega |(\xi_1 - h(\overline{N}))^+|^2 + \int_\Omega \nabla(\xi_1 - h(\overline{N}))^+(\xi_1 - h(\overline{N}))^+ dx),
\end{aligned}$$

and

$$\begin{aligned}
&\int_\Omega |\nabla(\xi_2 - h(\overline{N}))^+|^2 dx - \int_\Omega (g(\xi_1) - g(\xi_2))(\xi_2 - h(\overline{N}))^+ dx \\
\leq \;& C_2\gamma(\int_\Omega |(\xi_2 - h(\overline{N}))^+|^2 + \int_\Omega \nabla(\xi_2 - h(\overline{N}))^+(\xi_2 - h(\overline{N}))^+ dx),
\end{aligned}$$

from the definition of $\overline{N}$, we have

$$\int_\Omega (g(\xi_1) - g(\xi_2))(\xi_1 - h(\overline{N}))^+ dx - \int_\Omega (g(\xi_1) - g(\xi_2))(\xi_2 - h(\overline{N}))^+ dx \geq 0,$$

Hence, set $C_3 = \max(C_1, C_2)$, we can obtain

$$\begin{aligned}
&\int_\Omega |\nabla(\xi_1 - h(\overline{N}))^+|^2 dx + \int_\Omega |\nabla(\xi_2 - h(\overline{N}))^+|^2 dx \\
\leq \;& C_3\gamma(\int_\Omega |(\xi_1 - h(\overline{N}))^+|^2 + \int_\Omega \nabla(\xi_1 - h(\overline{N}))^+(\xi_1 - h(\overline{N}))^+ dx \\
&+ \int_\Omega |(\xi_2 - h(\overline{N}))^+|^2 + \int_\Omega \nabla(\xi_2 - h(\overline{N}))^+(\xi_2 - h(\overline{N}))^+ dx),
\end{aligned}$$

Setting $\gamma \leq \gamma_1 = \min(1, \frac{1}{C_3})$ and using Poincare inequality, we have

$$\|\nabla(\xi_1 - h(\overline{N}))^+\|_2^2 + \|\nabla(\xi_2 - h(\overline{N}))^+\|_2^2 \leq 0.$$

Therefore, we conclude $\xi_1, \xi_2 \leq h(\overline{N})$. Applying the monotonicity of h, we have $n_1, n_2 \leq \overline{N}$.

In the similar way, we can prove $n_1, n_2 \geq \underline{N}$. Finally, from the $W^{2,q}$- theory of the second-order elliptic equations in [26], we have

$$\|(\xi_1, \xi_2)\|_{W^{2,q}(\Omega)} \leq C(\overline{N}, \underline{N}, \varepsilon N, \mu N, \|h(n_{1D})\|_{L^q}, \|h(n_{2D})\|_{L^q}, \gamma, \|\nabla(\frac{1}{\tau_0})\|_{L^q(\Omega)}).$$

Taking μ_1, ε_1 small enough and for $\gamma \leq \gamma_1$, we have

$$\|(\xi_1, \xi_2)\|_{W^{2,q}(\Omega)} \leq C(\overline{N}, \underline{N}, 1, \|h(n_{1D})\|_{L^q}, \|h(n_{2D})\|_{L^q}, \|\nabla(\frac{1}{\tau_0})\|_{L^q(\Omega)}).$$

Because of the continuous imbedding $W^{2,q}(\Omega) \to C^{1,\delta}(\overline{\Omega})$, we can choose proper N such that

$$\|(n_1, n_2)\|_{C^{1,\delta}(\overline{\Omega})} \leq N. \tag{3.6}$$

This completes the proof.

Next, we are concerned with the analysis (3.3). The existence of a unique solution in $W^{2,2q}(\Omega)$ is immediately obtained if $\varepsilon \text{div} v_1 + \frac{1}{\tau_0} \geq 0$ and $\mu \text{div} v_2 + \frac{1}{\tau_0} \geq 0$, which hold $\frac{1}{\tau_0} \geq \max(\varepsilon_1, \mu_1)\gamma$. In the following Lemma we will give the estimate of (w_1, w_2) and then show $(u_1, u_2) \in B$.

Lemma 3.2. Let the assumptions (H1)-(H5) hold, then there exists $\gamma_2 > 0$ such that the solution (w_1, w_2) of (3.3) satisfies

$$\alpha^2 \|\Delta w_1\|_{L^{2q}(\Omega)} + \|w_1\|_{L^{2q}(\Omega)} \leq C(\varepsilon_1, \gamma_2)\gamma(\tau_4 + \tau_3\gamma)(1 + N), \tag{3.7}$$

$$\alpha^2 \|\Delta w_2\|_{L^{2q}(\Omega)} + \|w_2\|_{L^{2q}(\Omega)} \leq C(\mu_1, \gamma_2)\gamma(\tau_4 + \tau_3\gamma)(1 + N), \tag{3.8}$$

for all $\varepsilon \in [0, \varepsilon_1]$, $\mu \in [0, \mu_1]$, $\gamma \in [0, \gamma_2]$, $(m_1, m_2) \in A$ and $(v_1, v_2) \in B$. Furthermore, there exist $\gamma_3 > 0$ such that $(u_1, u_2) \in B$ for all $\gamma \in [0, \gamma_3]$.

proof. We firstly deduce (3.7) and then we can get (3.8) similarly. Multiplying the equation $(3.3)_1$ by w_1^{2q-1} and integrating by parts, we obtain

$$\begin{aligned} &(2q-1)\alpha^2 \int_\Omega |\nabla w_1|^2 |w_1|^{2q-2} dx + \int_\Omega (\frac{1}{\tau_0} + \varepsilon \frac{2q-1}{2q} \text{div} v_1) |w_1|^{2q} dx \\ = &-\int_\Omega \nabla(\frac{1}{\tau_0}) \cdot v_{1\perp} w_1^{2q-1} dx. \end{aligned}$$

Set $\gamma_2 = \frac{1}{2\tau_2 \epsilon}$, then $\frac{1}{\tau_0} + \varepsilon \frac{2q-1}{2q} \text{div} v_1 \geq \frac{1}{2\tau_2}$ holds for all $\gamma \in [0, \gamma_2]$, then we have

$$\frac{1}{2\tau_2} \|w_1\|^{2q}_{L^{2q}(\Omega)} \leq C\gamma \|\nabla(\frac{1}{\tau_0})\|_{L^{2q}(\Omega)} \|w_1\|^{2q-1}_{L^{2q}(\Omega)},$$

hence

$$\|w_1\|_{L^{2q}(\Omega)} \leq C\gamma \|\nabla(\frac{1}{\tau_0})\|_{L^{2q}(\Omega)}.$$

Further, using $(3.3)_3$ and the estimate (3.6), we can obtain (3.7). We now estimate on (u_1, u_2) from (3.4). From (3.4) we have

$$\|\mu_i\|_{W^{2,2q}(\Omega)} \leq C \|w_i\|_{L^{2q}(\Omega)},$$

consequently, (3.7) and (3.8) give

$$\|\sigma_i\|_{W^{1,2}(\Omega)} \leq C\gamma(1 + N)(\tau_4 + \tau_3\gamma). \tag{3.9}$$

Similarly, by using the estimate on $\alpha^2 \|\Delta w_i\|_{L^{2q}(\Omega)}$, we can obtain

$$\|\sigma_i\|_{W^{3,2q}(\Omega)} \leq C(\alpha, \gamma, N) \tag{3.10}$$

where $i = 1, 2$, which will omit in the proof for convenience.

The Neumann problem (3.5) has unique solution ψ_i since $\mathrm{div}\sigma_i = 0$ and since $\int_{\partial\Omega} n_{iD}u_{iD}dx = 0$ hold. We obtain the $W^{1,2q}(\Omega)$-estimate for $\nabla\psi_i$ just as for Dirichlet problems:

$$\|\nabla\psi_i\|_{W^{1,2q}(\Omega)} \le C(N)(\|u_{iD}\|_{W^{1,2q}(\Omega)} + \|\sigma_i\|_{L^{2q}(\Omega)}), \tag{3.11}$$

and by using (3.6) and (3.9)

$$\|\Delta\psi_i\|_{L^\infty(\Omega)} \le C(N)(\|\nabla\psi_i\|_{W^{1,2q}(\Omega)} + \|\sigma_i\|_{W^{1,2q}(\Omega)}).$$

Using the similar method, we obtain

$$\|\nabla\psi_i\|_{W^{2,2q}(\Omega)} \le C(N,\gamma,\epsilon)(\|u_{iD}\|_{W^{2,2q}(\Omega)} + \|\sigma_i\|_{W^{1,2q}(\Omega)}), \tag{3.12}$$

and

$$\|\Delta\psi_i\|_{W^{1,q}(\Omega)} \le C(N,\gamma,\epsilon)(\|\nabla\psi_i\|_{W^{1,2q}(\Omega)} + \|\sigma_i\|_{W^{1,2q}(\Omega)}).$$

(see [27] for $W^{2,p}$-estimate of solution of elliptic Neumann problems). We conclude from (3.9), (3.11) and by using $\mathrm{div}u_i = -\Delta\psi_i$:

$$\begin{aligned} &\|u_i\|_{W^{1,q}(\Omega)} \le C_4(N)(\gamma(\tau_4 + \tau_3\gamma) + \|u_{iD}\|_{W^{1,2q}(\Omega)}), \\ &\|\mathrm{div}u_i\|_{W^{1,q}(\Omega)} \le C_4(N)(\gamma(\tau_4 + \tau_3\gamma) + \|u_{iD}\|_{W^{1,2q}(\Omega)}). \end{aligned} \tag{3.13}$$

Finally, we choose $\gamma_3 = \min(\gamma_1, \gamma_2, 1/(3C_4(N)\tau_3))$, where γ_1, γ_2 are given by Lemmas 3.1 and 3.2 respectively. Then, if

$$\tau_4 \le \frac{1}{3C_4(N)\tau_3}, \ \|u_{iD}\|_{W^{1,2q}(\Omega)} \le \frac{\gamma}{3C_4(N)\tau_3},$$

hold, we have

$$\|u_i\|_{W^{1,q}(\Omega)} \le \gamma, \ \|\mathrm{div}u_i\|_{L^\infty(\Omega)} \le \gamma.$$

Thus, with the above choices of $\underline{N}, \overline{N}, N, \gamma$ we have

$$T : A \times B \to A \times B.$$

This completes the proof. Now we can show that

$$T : A \times B \to A \times B$$

is a continuous compact operator. At first, (3.10), (3.12) and (3.13) give

$$\|u_i\|_{W^{2,q}(\Omega)} \le C(\epsilon,\alpha), \ \|\mathrm{div}u_i\|_{W^{1,q}(\Omega)} \le C(\epsilon,\alpha), \tag{3.14}$$

which together with (3.6), implies that the image of $A \times B$ under T is precompact in $C^{0,1}(\overline{\Omega}) \times C^{0,1}(\overline{\Omega}) \times (W^{1,2q}(\Omega) \cap \{u_1 : \mathrm{div}u_1 \in L^\infty(\Omega)\}) \times (W^{1,2q}(\Omega) \cap \{u_2 : \mathrm{div}u_2 \in L^\infty(\Omega)\})$. Using $W^{2,q}$-estimates for elliptic boundary value problems, the proof of the continuity of T is standard and will be omitted here.

Finally let us give the proof of Theorem 1.1 as follows.

The proof of Theorem 1.1. From what we have showed that the regularized problem (2.1)-(2.2) has a solution $(n_1^\alpha, n_2^\alpha, u_1^\alpha, u_2^\alpha, \phi^\alpha) \in A \times B \times C^{1,\delta}(\overline{\Omega})$.

Now take a sequence $\alpha \to 0^+$. Then there is a subsequence, which we shall denote by the same symbol, such that

$$\begin{aligned} &n_1^\alpha \to n_1 \text{ in } W^{2,q}(\Omega) \text{ weakly},\\ &n_2^\alpha \to n_2 \text{ in } W^{2,q}(\Omega) \text{ weakly},\\ &u_1^\alpha \to u_1 \text{ in } W^{1,2q}(\Omega) \text{ weakly},\\ &u_2^\alpha \to u_2 \text{ in } W^{1,2q}(\Omega) \text{ weakly},\\ &\phi^\alpha \to \phi \text{ in } C^{1,\delta}(\overline{\Omega}) \text{ weakly}, \end{aligned}$$

hold for solutions $(n_1^\alpha, n_2^\alpha, u_1^\alpha, u_1^\alpha, \phi^\alpha)$ constructed above. Then, we can obviously obtain that $(n_1, n_2, u_1, u_2, \phi)$ satisfies (1.2)-(1.3). This completes the proof.

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Competing Interests

Authors have declared that no competing interests exist.

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