



## Revisiting Bohr's Theory via a Relationship between Magnetic Constant and Bohr Radius of Any Element

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### Author's contribution

The sole author designed, analyzed, interpreted and prepared the manuscript.

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### ABSTRACT

**Objectives:** The purposes of this research are 1<sup>st</sup>, to show that there is a relationship between magnetic constant and Bohr radius for any atom and 2<sup>nd</sup>, to verify this relationship with illustrative calculations.

**Place and Duration of Study:** Department of Chemistry and Biochemistry, Research Division, Ude International Concepts LTD (862217), B. B. Agbor, Delta State, Nigeria; Owa Alizomor Secondary School, Owa Alizomor, Ika North East, Delta State, Nigeria.

**Methods:** Theoretical and calculational.

**Results:** The result of derived equations for Bohr radius for any atom as they relate to magnetic constant ( $\mu_0$ ) and effective nuclear charge ( $Z_{\text{eff}}$ ) are as shown below where  $h$ ,  $n$ , and  $c$ , are Planck's constant, principal quantum number, and velocity of light in a vacuum respectively;  $E$ ,  $E_H$ ,  $e$ , and  $m_e$  are average ionization energy (IE) for atom other than hydrogen, IE for hydrogen, charge of an electron, and mass of an electron respectively. Using the Bohr's radius for hydrogen as an example the radius,  $a$ , obtainable from all equations are similar being  $\approx 5.3 \exp(-11) \text{ m}$ . 
$$a = \frac{nh}{\pi} \cdot \frac{2}{\sqrt{8Em_e}}$$

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(From literature [12])  $a = \frac{1}{m_e} \cdot \sqrt{\frac{E_H}{E}} \cdot \frac{h^2 n}{\mu_0 c^2 e^2 \pi}$  (From this research). The derived equation for effective nuclear charge ( $Z_{\text{eff}}$ ) is  $Z_{\text{eff}} = \sqrt{\frac{8E}{m_e} \cdot \frac{nh}{c^2 e^2 \mu_0}}$  and using hydrogen as example,  $Z_{\text{eff}} \cong 1$ .

**Conclusion:** The results showed that magnetic constant like fine structure constant can be separately related to the radius of elements. Several pieces of evidence of this issue expressed via different equations may justify or validate Bohr's classical model. If the mathematical equations can reproduce the parameters, 'a' and  $Z_{\text{eff}}$  for hydrogen, multi-electron atoms could not be the exception. Reproducing 'a' and  $Z_{\text{eff}}$  for hydrogen and by extension every hydrogenic atom (but not limited to hydrogenic atom as reported in the literature) seem to validate the equations in this research and Bohr's original equation in particular.

*Keywords: Magnetic constant; fine structure constant; Bohr radius; effective nuclear charge; hydrogenic atom; non-hydrogenic atom.*

## 1. INTRODUCTION

This research focuses on the relationship or interconnection between fundamental physical constants. Therefore, the work of Chakeres [1] is worth considering in this research; relationships called ratio relationships between the fine structure constant, apart from other parameters, Bohr radius, the ionization energy of hydrogen and classical electron radius were reviewed by the author [1]. Of paramount importance and interest is the observation that the dimensionless product of  $c$ ,  $\mu_0$ , and  $e^2$ , divided by  $h$  is equal to  $2\alpha$  [1] where,  $\alpha$ ,  $h$ ,  $c$ ,  $\mu_0$ , and  $e$  are the fine structure constant, Planck's constant, velocity of light in a vacuum, magnetic constant, and charge of an electron.

The paper by Lush [2] is highly specialized one, comprehensible to core physicist who is neck – deep in the field and not for those at the periphery. According to Lush [2], Bohr's model of atomic hydrogen is a modification of prior Rutherford model with an *ad hoc* introduction of a quantum principle. The quantum principle in Bohr's model is that stable electron orbits are those with angular momentum in integer multiples of reduced Planck constant,  $\hbar$ . The model explained the low resolution emission spectrum of hydrogen and, extension from circular to elliptical orbits enable the explanation or perhaps derivation of fine structure constant [2]. An invalid model, if this is the concerning Sommerfeld elliptical model, cannot give a universally known constant like fine structure constant,  $\alpha$ .

However, the unsuitability of these theories for multi-electron atoms led to the emergence of Heisenberg and Schrödinger postulations [1] that

are criticized respectively [3] as follows: Heisenberg uncertainty principle has been seen to be an invalid physical description of the electron because according to Mill [3], the wave function is interpreted as the probability of the position of the electron which puts it everywhere at once with an infinite number of positions and energies simultaneously including ones with negative kinetic energies. Schrödinger's theory have been criticized for failing to predict electron spin and leads to models with nonsensical consequences such as negative energy states of the vacuum, infinities, and negative kinetic energy (KE) [3]. It is not certain if there is a machine or device that can convert negative KE to useful work. Thus contrary to the position that the validity of the model for one electron (or hydrogenic) atom was investigated by invoking the Heisenberg principle and Schrodinger-Dirac formalism [4], two different approaches, the deterministic approach of Bohr and stochastic approach that characterizes Heisenberg principle and Schrödinger-Dirac formalism are 'strange bedfellows'. This position is supported by the assertion that Schrödinger formalism unlike Pauli Exclusion Principle, cannot explain chemical periodicity [4]. Recent development shows that by relating fine structure constant,  $\alpha$ , to the key periodic properties of elements, not just hydrogen atom or hydrogenic ions but all multi-electron atoms whose first ionization energies (IE) (but not limited to 1<sup>st</sup> IE), for instance, are known, the original Bohr theory seems to be justified for ground state atoms and ions. The scope of the models or equations to be derived are clearly defined by the fact that an equation relating magnetic constant to Bohr's radius and effective nuclear charge ( $Z_{\text{eff}}$ ) can be used to calculate atomic radius otherwise called

Bohr's radius and also  $Z_{\text{eff}}$  for any element. Thus given that  $\alpha$  has been related to the periodic properties of elements, there is a motivation to determine as part of the objectives of this research, the relationship between magnetic constant and the radius of all elements that fall within the known 103 elements and with  $Z_{\text{eff}}$  and to illustrate this relationship with calculated examples.

## 2. BRIEF THEORETICAL BACKGROUND

### 2.1 Meaning of Fine Structure Constant

Beginning from known (the established relationship between  $\alpha$  and periodic properties) to the unknown, there is need to state what Oldershaw [5] called the conventional mathematical definition of the fine structure constant,  $\alpha$  and after that, its 'qualitative or conceptual definition' as follows:

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} \quad (1)$$

Where  $e, \epsilon_0, \hbar$ , and  $c$  are the charge of an electron, permittivity in free space, reduced Planck constant and velocity of light in a vacuum. Since  $\alpha$  is a dimensionless parameter, it should be a ratio of 2 quantities with the same dimensionality [5], similar to  $\pi$ . The product,  $\hbar c$ , is related to revised Planck mass,  $M_{\text{pl}}$  [5], as follows:

$$\hbar c = G_{-1}(M_{\text{pl}})^2 \quad (2)$$

Substituting into the re-grouped Eq. (1) gives:

$$\alpha = \frac{\left[\frac{e^2}{4\pi\epsilon_0}\right]}{\left[G_{-1}(M_{\text{pl}})^2\right]} \quad (3)$$

The numerator of Eq. (3) [5] is the square of the unit electromagnetic charge (which can be interpreted as the strength of the unit of electromagnetic interaction) and the denominator is the square of the unit gravitational "change" for atomic scale system (which can be interpreted as the strength of the unit gravitational interaction for atomic scale system). Thus according to Oldershaw [5], the answer to the meaning of the parameter,  $\alpha$  is:- It is the ratio of the strengths of the fundamental unit of electromagnetic and gravitational interactions. This is contrary to Dattoli's [6] and, Heyrovska and Narayan [7] position about the mystical nature of  $\alpha$  but similar

to the finding by Udema [8] who derived the following relationship:

$$a \cong \frac{1}{137} \frac{nh^2\epsilon_0 c}{\pi e^2 \sqrt{2m_e E_1}} \quad (4)$$

Where  $\alpha = \frac{1}{137} m_e, h$ , and  $E_1$  are the mass of an electron, Planck's constant and first average ionization energy;  $a$  is Bohr radius for any atom. However, if in place of  $1/137$ , which has different ways of derivation,  $e^2/2\epsilon_0\hbar c$  is substituted into Eq. (4),  $a \cong \frac{nh}{2\pi^2\sqrt{2m_e E_1}}$  is obtained. The fine

structure constant is also defined as:  $\frac{\lambda_{\text{C,H}}}{\lambda_{\text{dB}}} = \frac{1}{137.03} = \frac{2.627}{360}$  [7] "where  $\lambda_{\text{C,H}}$ , the Compton wavelength for hydrogen is a distance equivalent to an arc length,  $\lambda_{\text{dB}}$  ( $= 2\pi a_{\text{B}}$ ), on the circumference of a circle where the Bohr radius is  $a_{\text{B}}$ " and the Golden ratio  $\phi$  is related to  $\alpha$  according to the equation:  $\frac{\phi^2}{360} = \frac{2.618}{360} = \frac{1}{137.508}$ . The fine structure constant, is also part of the Rydberg constant,  $R_{\infty} = \frac{\alpha^2}{2\lambda_e}$  (half the square of fine structure constant divide by Compton wavelength of the electron) [9].

The fine structure constant,  $\alpha$ , has also been implicated in the relationship between the two electric charges  $Q$  and  $-Q$  of an unclear model, the new composite superluminal double-helix photon model and the electron's charge,  $e$  such

that  $Q = e\sqrt{\frac{2}{\alpha}} = 16.6e$ . In such relationship,  $\alpha$  is defined as a measure of the strength of interaction between an electron and a photon in an unfamiliar field of quantum electrodynamics (QED) [10]. On the other hand, the permeability of free space ( $\mu_0$ ), is according to the Committee on Data for Science and Technology (CODATA) also known as the magnetic constant ( $12.566 370 614 \times 10^{-7} \text{ N A}^{-2}$ ), and is a measure of the amount of resistance encountered when forming a magnetic field in a classical vacuum (Wikipedia). This research is significant because it serves to demystify the fundamental physical constant, the fine structure constant in particular and with another fundamental physical constant,  $\mu_0$  as specified by CODATA, it can be used to determine separately the periodic properties of elements. Periodic properties such as ionization energy and dependent cognate parameter, the effective nuclear charge could aid the characterization of chemical reactions between elements and between compounds or between

compounds and elements. According to Ghosh and Biwas [11] such properties can enhance the comprehension, elucidation, correlation, prediction, and even calculation of many size dependent physico-chemical properties of atoms and ions.

## 2.2 Mathematical Model Linking Bohr's Radius for Any Atom with Magnetic Constant and Velocity of Light

The mathematical relationship between Bohr radius, for any atom, and magnetic constant, needs to be derived in two steps.

### 2.2.1 Derivation of the relationship between effective nuclear charge and magnetic constant and velocity of light

To begin with, there is need to recall mathematical definition of fine structure constant regarding magnetic constant and the velocity of light in a vacuum [1].

$$1/\alpha = \frac{2h}{c\mu_0 e^2} \quad (5)$$

Also as in any standard text book [12] and as applied in other literature [8] is another expression for the fine structure constant,  $\alpha = \frac{e^2}{2\varepsilon_0 ch}$ . However, in order to relate the ionization energy to other physical constants variable parameters such as Bohr radius for any atom, Eq. (5) is related to  $c/\sqrt{\frac{2E_H}{m_e}}$  (where  $E_H$  is the average ionization energy of hydrogen) as follows:

$$\frac{1}{\alpha} = \frac{2h}{c\mu_0 e^2} = \frac{c}{2\sqrt{\frac{2E_H}{m_e}}} \quad (6)$$

Making  $c$  subject of the formula in Eq. (6) gives.

$$c = \sqrt[4]{\frac{8E_H}{m_e}} \cdot 2\sqrt{\frac{h}{\mu_0}} \cdot \frac{1}{e} \quad (7)$$

As in literature [8],  $E_H = \frac{n^2 E}{Z_{\text{eff}}^2}$ .  $E$  and  $Z_{\text{eff}}$  are the average ionization of any atom other than any hydrogenic atom and effective nuclear charge respectively. Substitution of this into Eq. (7) gives:

$$c = \sqrt[4]{\frac{8n^2 E}{Z_{\text{eff}}^2 m_e}} \cdot 2\sqrt{\frac{h}{\mu_0}} \cdot \frac{1}{e} \quad (8)$$

Making the  $Z_{\text{eff}}^2$  subject of the formula in Eq. (8) and taking the square root gives:

$$Z_{\text{eff}}^2 = \frac{8n^2 E h^2}{m_e c^4 \mu_0^2 e^4} \quad (9)$$

$$Z_{\text{eff}} = \sqrt[2]{\frac{8E}{m_e}} \cdot \frac{nh}{c^2 e^2 \mu_0} \quad (10)$$

### 2.2.2 Relating Bohr's radius with the velocity of light and the magnetic constant

Meanwhile, Bohr's original mathematical model for the radius ( $a$ ) of any atom other than hydrogenic atom is:

$$a = \frac{n^2 h^2 \varepsilon_0}{\pi m_e e^2 Z_{\text{eff}}} \quad (11)$$

Substitution of Eq. (10) into Eq. (11) should give:

$$\begin{aligned} a &= \frac{n^2 h^2 \varepsilon_0}{\pi m_e e^2} \cdot \sqrt[2]{\frac{m_e}{8E}} \cdot \frac{c^2 e^2 \mu_0}{nh} \\ &= \frac{nh \varepsilon_0 \mu_0 c^2}{\pi} \cdot \sqrt[2]{\frac{1}{8Em_e}} \end{aligned} \quad (12a)$$

Substitution of relevant parameters into Eq. (12a) gives the same results as the results obtained from substitution into the derived equation ( $a = \frac{nh}{\pi^2 \sqrt{8m_e E}}$ ) elsewhere [13]. This is the case because  $\varepsilon_0 \mu_0 c^2 = 1$ . Therefore, from the latter,  $\varepsilon_0$  can be derived to give:

$$\varepsilon_0 = 1/\mu_0 c^2 \quad (12b)$$

Also, in the past [8], it has been shown that fine structure constant in Eq. (4) or as may be determined by other method or definition can be used to determine Bohr's radius of any atom. If one word is good enough for the scientist, then few selected elements including the reference element hydrogen, may be sufficient to test the validity of Eq. (12a) and ultimately Bohr's mathematical model for any atom in its ground state.

The 1<sup>st</sup> step in the derivation of a relationship between Bohr's radius, velocity of light in a vacuum, and magnetic constant is by recognizing that in Eq. (12a) are, with the exception of  $e$ , fundamental constants that can be found in the equation of fine structure constant ( $\alpha = \frac{e^2}{2\varepsilon_0 ch}$ ) given that  $E$  is experimentally determined. By including those constants including  $e^2$  and 2, as nominators and denominators in Eq. (12a), one can obtain,

$$a = \frac{2e^2nh^2\epsilon_0^2\mu_0c^3}{2e^2h\epsilon_0\pi c} \cdot \sqrt[2]{\frac{1}{8Em_e}} \quad (13)$$

Regrouping of appropriate parameters in Eq. (13) gives:

$$a = \frac{e^2}{2\epsilon_0ch} \cdot \frac{2\epsilon_0^2\mu_0c^3h^2n}{e^2\pi} \cdot \sqrt[2]{\frac{1}{8Em_e}} \quad (14a)$$

Therefore,

$$a = \alpha \cdot \frac{2\epsilon_0^2\mu_0c^3h^2n}{e^2\pi} \cdot \sqrt[2]{\frac{1}{8Em_e}} \quad (14b)$$

However, simplification of Eq. (14a) and (14b) leads to the equation:  $a = \frac{nh}{\pi^2\sqrt{8m_eE}}$ . Therefore, the 2<sup>nd</sup> step is to substitute the reciprocal of the far right hand side of Eq. (6) into Eq. (14b) to give 1<sup>st</sup>:

$$a = \frac{2}{c} \cdot \sqrt{\frac{2E_H}{m_e}} \cdot \frac{\epsilon_0^2\mu_0c^3h^2n}{e^2\pi} \cdot \sqrt[2]{\frac{1}{8Em_e}} \quad (15a)$$

Then, after rearrangement and simplification of Eq. (15a), the result is:

$$a = \frac{1}{m_e} \cdot \sqrt{\frac{E_H}{E}} \cdot \frac{\epsilon_0^2\mu_0c^2h^2n}{e^2\pi} \quad (15b)$$

Next, the square of Eq. (12b) need to be taken and substituted into Eq. (15a) and Eq. (15b) to give respectively the following:

$$a = \frac{2}{c} \cdot \sqrt{\frac{2E_H}{m_e}} \cdot \frac{h^2n}{\mu_0ce^2\pi} \cdot \sqrt[2]{\frac{1}{8Em_e}} \quad (16a)$$

$$a = \frac{1}{m_e} \cdot \sqrt{\frac{E_H}{E}} \cdot \frac{h^2n}{\mu_0c^2e^2\pi} \quad (16b)$$

Equation (16b) relates Bohr's radius for any atom with the square root of the ratio of the average ionization energy of hydrogen to average ionization of another element. It is important to realize that if  $E_H = E$ ,  $a$ , should be  $\cong 5.29 \exp(-11) \text{ m}$ , as long as  $n = 1$  in such situation.

### 3. MATERIALS AND METHODS

The methods are purely theoretical in which spectroscopic data, average 1<sup>st</sup> ionization energies of different elements are substituted into derived equations, Eq. (10) for  $Z_{\text{eff}}$  and Eq. (16b) for Bohr's radius,  $a$ . The fine structure constant,  $\alpha$ , and the magnetic constant,  $\mu_0$  are

brought into the same equation (Eq. (14b)) from which the average ionization energy for hydrogen, hydrogenic atom and non-hydrogenic atoms can be related (Eq. (16b)).

### 4. RESULTS AND DISCUSSION

Every attempt was made to derive equations that can be used to calculate the effective nuclear charge,  $Z_{\text{eff}}$  (Eq. (10)) and Bohr's radius for any atom (Eq. (16b)). All equations were related to the magnetic constant. The values of  $Z_{\text{eff}}$  and  $a$  (Table1) obtainable by substituting physical constants and 1<sup>st</sup> average ionization energies [14] into the relevant equations in this research are the same as those reported elsewhere [8,13]. It must be stated however, that the radii obtained in this research and in the past [8,13] differ from those obtained by Owolabi et al. [15], Ghosh and Biswas [11,16]. Consequently the effective nuclear charges determined by substituting spectroscopically determined ionization energies into relevant equations differ widely for all elements except hydrogen (Table 1). This is exactly the case when atomic radii of 103 elements from literature [17] were substituted into well known equation ( $Z_{\text{eff}} = \frac{n^2h^2\epsilon_0}{\pi m a e^2}$ ). The implication however, is that, when properties such as  $Z_{\text{eff}}$  and  $a$  are determined by other approaches, and the values substituted into appropriate equations in this research and elsewhere [8,13], the values of  $\alpha$  and  $\mu_0$  can be calculated precisely unlike in one instance in literature [18].

While the report that the opacity of suspended graphene (one atom thick) is defined solely by  $\alpha$  which also describes coupling between light and relativistic electrons and is traditionally associated with QED rather than material science [18] may be validly clear to core physicist, but it is rather strange to suggest that,  $\alpha = \frac{e^2}{\hbar c} \approx [18]$ . Rather,  $\alpha = \frac{e^2}{\hbar c} = 8.122 \exp(-13)$  as long as  $\hbar$ ,  $e$ , and  $c$  are the reduced Planck's constant, electron charge, and velocity of light in a vacuum. Another concern is about the equation for Bohr radius expressed as [2]  $R_B$  (which is  $a$  in this research) =  $\frac{\hbar^2}{m_e e^2} \approx 5.3 \times \exp(-09) \text{ cm}$ . The result should rather be 0.4756 without the unit of length. The appropriate equation where reduced Planck constant is the case should be:

$$R_B = \frac{4\pi\hbar^2\epsilon_0}{m_e e^2} \cong 5.3 \times \exp(-11) \text{ m} \quad (17)$$

Table 1. The effective nuclear charge and corresponding 1<sup>st</sup> radii of selected elements

Element	Ground state electronic configuration	1 <sup>st</sup> effective nuclear charge (From this research)	1 <sup>st</sup> Bohr's radius/Å (From this research)	1 <sup>st</sup> effective nuclear charge using data from Islam and Ghosh [17] for the determination	1 <sup>st</sup> Bohr's radius/Å from Islam and Ghosh [17]
Hydrogen	1s <sup>1</sup>	~ 1.00	0.53	~ 1	~ 0.529
Helium	1s <sup>2</sup>	1.34	0.39	1.82	0.291
Lithium	[He] 2s <sup>1</sup>	1.26	1.68	1.60	1.323
Beryllium	[He] 2s <sup>2</sup>	1.66	1.28	2.78	0.762
Boron	[He] 2s <sup>2</sup> 2p <sup>1</sup>	1.56	1.36	2.47	0.857
Carbon	[He] 2s <sup>2</sup> 2p <sup>2</sup>	1.82	1.16	3.36	0.630
Nitrogen	[He] 2s <sup>2</sup> 2p <sup>3</sup>	2.07	1.02	4.35	0.487
Oxygen	[He] 2s <sup>2</sup> 2p <sup>4</sup>	2.00	1.06	4.09	0.518
Fluorine	[He] 2s <sup>2</sup> 2p <sup>5</sup>	2.26	0.94	5.27	0.402
Neon	[He] 2s <sup>2</sup> 2p <sup>6</sup>	2.51	0.84	6.45	0.328
Sodium	[Ne] 3s <sup>1</sup>	1.84	2.58	3.45	1.381
Magnesium	[Ne] 3s <sup>2</sup>	2.25	2.12	5.14	0.926
Aluminum	[Ne] 3s <sup>2</sup> 3p <sup>1</sup>	1.99	2.39	4.07	1.169
Silicon	[Ne] 3s <sup>2</sup> 3p <sup>2</sup>	2.32	2.05	5.49	0.868
Phosphorus	[Ne] 3s <sup>2</sup> 3p <sup>3</sup>	2.63	1.81	7.03	0.677
Sulphur	[Ne] 3s <sup>2</sup> 3p <sup>4</sup>	2.62	1.82	7.03	0.677
Chlorine	[Ne] 3s <sup>2</sup> 3p <sup>5</sup>	2.93	1.63	8.74	0.545
Argon	[Ne] 3s <sup>2</sup> 3p <sup>6</sup>	3.23	1.47	10.73	0.444
Potassium	[Ar] 4s <sup>1</sup>	1.69	3.75	5.15	1.645
Calcium	[Ar] 4s <sup>2</sup>	2.68	3.16	7.27	1.164
Scandium	[Ar] 3d <sup>1</sup> 4s <sup>1</sup>	2.78	3.05	7.81	1.084
Titanium	[Ar] 3d <sup>2</sup> 4s <sup>2</sup>	2.83	2.99	8.13	1.042
Vanadium	[Ar] 3d <sup>3</sup> 4s <sup>2</sup>	2.82	3.01	8.04	1.053
Chromium	[Ar] 3d <sup>5</sup> 4s <sup>1</sup>	2.82	3.00	7.97	1.047
Manganese	[Ar] 3d <sup>5</sup> 4s <sup>2</sup>	2.96	2.86	8.89	0.952
Iron	[Ar] 3d <sup>6</sup> 4s <sup>2</sup>	3.05	2.78	9.94	0.899
Cobalt	[Ar] 3d <sup>7</sup> 4s <sup>2</sup>	3.04	2.78	9.42	0.899
Nickel	[Ar] 3d <sup>8</sup> 4s <sup>2</sup>	~3.00	2.82	9.20	0.920

Element	Ground state electronic configuration	1 <sup>st</sup> effective nuclear charge (From this research)	1 <sup>st</sup> Bohr's radius/Å (From this research)	1 <sup>st</sup> effective nuclear charge using data from Islam and Ghosh [17] for the determination	1 <sup>st</sup> Bohr's radius/Å from Islam and Ghosh [17]
Copper	[Ar] 3d <sup>10</sup> 4s <sup>1</sup>	3.01	2.81	9.25	0.915
Zinc	[Ar] 3d <sup>10</sup> 4s <sup>2</sup>	3.32	2.55	11.20	0.756
Gallium	[Ar] 3d <sup>10</sup> 4s <sup>2</sup> 4p <sup>1</sup>	2.66	3.19	7.27	1.164
Germanium	[Ar] 3d <sup>10</sup> 4s <sup>2</sup> 4p <sup>2</sup>	3.05	2.78	9.59	0.883
Arsenic	[Ar] 3d <sup>10</sup> 4s <sup>2</sup> 4p <sup>3</sup>	3.40	2.49	11.78	0.719
Selenium	[Ar] 3d <sup>10</sup> 4s <sup>2</sup> 4p <sup>4</sup>	3.39	2.50	11.78	0.719
Bromine	[Ar] 3d <sup>10</sup> 4s <sup>2</sup> 4p <sup>5</sup>	3.73	2.27	14.30	0.592
Krypton	[Ar] 3d <sup>10</sup> 4s <sup>2</sup> 4p <sup>6</sup>	4.06	2.09	16.83	0.503
Rubidium	[Kr] 5s <sup>1</sup>	2.77	4.78	7.72	1.714
Strontium	[Kr] 5s <sup>2</sup>	3.23	4.09	10.54	1.254
Yttrium	[Kr] 4d <sup>1</sup> 5s <sup>2</sup>	3.38	3.91	11.52	1.148
Zirconium	[Kr] 4d <sup>2</sup> 5s <sup>2</sup>	3.40	3.79	12.32	1.074
Niobium	[Kr] 4d <sup>4</sup> 5s <sup>1</sup>	3.52	3.75	12.50	1.058
Molybdenum	[Kr] 4d <sup>5</sup> 5s <sup>1</sup>	3.61	3.67	13.16	1.005
Technetium	[Kr] 4d <sup>5</sup> 5s <sup>2</sup>	3.66	3.62	13.51	0.979
Ruthenium	[Kr] 4d <sup>7</sup> 5s <sup>1</sup>	3.68	3.60	13.67	0.968
Rhodium	[Kr] 4d <sup>8</sup> 5s <sup>1</sup>	3.70	3.57	13.90	0.952
Palladium	[Kr] 4d <sup>10</sup>	3.13	2.70	15.44	0.857
Silver	[Kr] 4d <sup>10</sup> 5s <sup>1</sup>	3.73	3.55	14.13	0.936
Cadmium	[Kr] 4d <sup>10</sup> 5s <sup>2</sup>	4.06	3.25	16.66	0.794
Indium	[Kr] 4d <sup>10</sup> 5s <sup>2</sup> 5p <sup>1</sup>	4.09	3.23	10.78	1.227
Tin	[Kr] 4d <sup>10</sup> 5s <sup>2</sup> 5p <sup>2</sup>	3.67	3.60	13.74	0.963
Antimony	[Kr] 4d <sup>10</sup> 5s <sup>2</sup> 5p <sup>3</sup>	3.99	3.32	16.04	0.825
Tellurium	[Kr] 4d <sup>10</sup> 5s <sup>2</sup> 5p <sup>4</sup>	4.07	3.25	16.90	0.783
Iodine	[Kr] 4d <sup>10</sup> 5s <sup>2</sup> 5p <sup>5</sup>	4.38	3.02	19.54	0.677
Xenon	[Kr] 4d <sup>10</sup> 5s <sup>2</sup> 5p <sup>6</sup>	4.72	2.80	22.73	0.582
Caesium	[Xe]6s <sup>1</sup>	3.21	5.93	10.35	1.841
Barium	[Xe]6s <sup>2</sup>	3.74	5.13	13.85	1.375
Lanthanum	[Xe]5d <sup>1</sup> 6s <sup>2</sup>	3.84	4.96	14.88	1.280
Cerium	[Xe]4f <sup>2</sup> 6s <sup>2</sup>	3.77	4.98	14.82	1.285

Element	Ground state electronic configuration	1 <sup>st</sup> effective nuclear charge (From this research)	1 <sup>st</sup> Bohr's radius/Å (From this research)	1 <sup>st</sup> effective nuclear charge using data from Islam and Ghosh [17] for the determination	1 <sup>st</sup> Bohr's radius/Å from Islam and Ghosh [17]
Praseodymium	[Xe]4f <sup>3</sup> 6s <sup>2</sup>	3.80	5.01	14.70	1.296
Neodymium	[Xe]4f <sup>4</sup> 6s <sup>2</sup>	3.82	4.98	14.58	1.307
Promethium	[Xe]4f <sup>5</sup> 6s <sup>2</sup>	3.79	4.95	15.13	1.259
Samarium	[Xe]4f <sup>6</sup> 6s <sup>2</sup>	3.85	4.90	15.39	1.238
Europium	[Xe]4f <sup>7</sup> 6s <sup>2</sup>	3.87	4.92	15.53	1.227
Gadolinium	[Xe]4f <sup>7</sup> 5d <sup>1</sup> 6s <sup>2</sup>	4.03	4.73	16.90	1.127
Terbium	[Xe]4f <sup>9</sup> 6s <sup>2</sup>	3.98	4.84	16.22	1.174
Dysprosium	[Xe]4f <sup>10</sup> 6s <sup>2</sup>	3.96	4.81	16.52	1.153
Holmium	[Xe]4f <sup>11</sup> 6s <sup>2</sup>	3.99	4.77	16.83	1.132
Erbium	[Xe]4f <sup>12</sup> 6s <sup>2</sup>	4.02	4.74	17.22	1.106
Thulium	[Xe]4f <sup>13</sup> 6s <sup>2</sup>	3.92	4.71	17.57	1.084
Ytterbium	[Xe]4f <sup>14</sup> 6s <sup>2</sup>	4.05	4.68	17.82	1.069
Lutetium	[Xe]4f <sup>14</sup> 5d <sup>1</sup> 6s <sup>2</sup>	4.03	5.03	15.80	12.06
Hafnium	[Xe]4f <sup>14</sup> 5d <sup>2</sup> 6s <sup>2</sup>	~4.30	4.48	19.68	0.968
Tantalum	[Xe]4f <sup>14</sup> 5d <sup>3</sup> 6s <sup>2</sup>	4.57	4.17	21.7	0.878
Tungsten	[Xe]4f <sup>14</sup> 5d <sup>4</sup> 6s <sup>2</sup>	~4.60	4.15	22.79	0.836
Rhenium	[Xe]4f <sup>14</sup> 5d <sup>5</sup> 6s <sup>2</sup>	4.56	4.17	22.79	0.836
Osmium	[Xe]4f <sup>14</sup> 5d <sup>6</sup> 6s <sup>2</sup>	5.65	3.97	24.67	0.772
Iridium	[Xe]4f <sup>14</sup> 5d <sup>7</sup> 6s <sup>2</sup>	4.91	3.89	26.28	0.725
Platinum	[Xe]4f <sup>14</sup> 5d <sup>9</sup> 6s <sup>2</sup>	4.88	3.90	25.71	0.741
Gold	[Xe]4f <sup>14</sup> 5d <sup>10</sup> 6s <sup>2</sup>	4.94	3.86	27.29	0.698
Mercury	[Xe]4f <sup>14</sup> 5d <sup>10</sup> 6s <sup>2</sup>	4.04	3.62	30.77	0.619
Thallium	[Xe]4f <sup>14</sup> 5d <sup>2</sup> 6s <sup>2</sup> 6p <sup>1</sup>	4.07	4.74	16.37	1.164
Lead	[Xe]4f <sup>14</sup> 5d <sup>2</sup> 6s <sup>2</sup> 6p <sup>2</sup>	4.43	4.27	19.91	0.957
Bismuth	[Xe]4f <sup>14</sup> 5d <sup>2</sup> 6s <sup>2</sup> 6p <sup>3</sup>	4.39	4.31	19.58	0.973
Polonium	[Xe]4f <sup>14</sup> 5d <sup>2</sup> 6s <sup>2</sup> 6p <sup>4</sup>	4.72	4.01	22.64	0.841
Astatine	[Xe]4f <sup>14</sup> 5d <sup>2</sup> 6s <sup>2</sup> 6p <sup>5</sup>	4.96	3.83	25.92	0.735
Radon	[Xe]4f <sup>14</sup> 5d <sup>2</sup> 6s <sup>2</sup> 6p <sup>6</sup>	5.33	3.55	28.82	0.661
Francium	[Fr]7s <sup>1</sup>	3.77	6.84	14.72	1.762
Radium	[Fr]7s <sup>2</sup>	4.36	5.95	19.07	1.360



Element	Ground state electronic configuration	1 <sup>st</sup> effective nuclear charge (From this research)	1 <sup>st</sup> Bohr's radius/Å (From this research)	1 <sup>st</sup> effective nuclear charge using data from Islam and Ghosh [17] for the determination	1 <sup>st</sup> Bohr's radius/Å from Islam and Ghosh [17]
Actinium	[Fr]6d <sup>1</sup> 7s <sup>2</sup>	4.31	6.01	18.71	1.386
Thorium	[Fr]6d <sup>2</sup> 7s <sup>2</sup>	4.68	5.54	22.80	1.137
Protactinium	[Fr]5f <sup>2</sup> 6d <sup>1</sup> 7s <sup>2</sup>	4.60	5.63	21.41	1.211
Uranium	[Fr]5f <sup>3</sup> 6d <sup>1</sup> 7s <sup>2</sup>	4.72	5.49	22.59	1.148
Neptunium	[Fr]5f <sup>4</sup> 6d <sup>1</sup> 7s <sup>2</sup>	4.75	5.46	22.91	1.132
Plutonium	[Fr]5f <sup>6</sup> 7s <sup>1</sup>	4.67	5.55	22.18	1.169
Americium	[Fr]5f <sup>7</sup> 7s <sup>2</sup>	4.64	5.58	21.97	1.180
Curium	[Fr]5f <sup>7</sup> 6d <sup>1</sup> 7s <sup>2</sup>	4.66	5.57	22.09	1.174
Berkelium	[Fr]5f <sup>9</sup> 7s <sup>2</sup>	4.74	5.48	23.01	1.127
Californium	[Fr]5f <sup>10</sup> 7s <sup>2</sup>	4.72	5.44	23.44	1.106
Einsteinium	[Fr]5f <sup>11</sup> 7s <sup>2</sup>	4.81	5.44	24.03	1.079
Fermium	[Fr]5f <sup>12</sup> 7s <sup>2</sup>	4.84	5.36	24.39	1.063
Mendelevium	[Fr]5f <sup>13</sup> 7s <sup>2</sup>	4.87	5.33	24.88	1.042
Nobelium	[Fr]5f <sup>14</sup> 7s <sup>2</sup>	4.89	5.30	25.27	1.026
Lawrencium	[Fr]5f <sup>11</sup> 6d <sup>1</sup> 7s <sup>2</sup>	4.19	6.19	17.89	1.449

The values of ionization energy per mole for each of the elements in the table can be found in literature cited [13]. For the purpose of confirmation, the 1<sup>st</sup> ionization energies per mole, for hydrogen and sodium are 1312 and 496 kJ/mol respectively; calculated parameters are approximated to 2 decimal. The atomic radii obtained from literature [17]

were substituted into the equation:  $Z_{eff} = \frac{n^2 h^2 \epsilon_0}{\pi m a e^2}$

At this juncture, it is important not to shy away from the fact that there has been criticism of Bohr's theory by some scholars [19,20]. However, whatever the criticism might be, the modern approach has also its modest share of criticism by a no-nonsense scientist [3]. Incidentally there is a view that "in the present day where the essentiality of quantum behaviour being nonlocal and nondeterministic has been directly challenged and based on the results described, it may be warranted to re-examine the potential for relativistic classical physics to describe phenomena that were hitherto thought to be purely and fundamentally quantum mechanical in nature" [2]. This opinion is in support of the informed opinion by Mills [3] who has sufficient mathematical knowledge (herein referred to as "equipment") to challenge Schrodinger-Dirac and Heisenberg principle unlike in this research.

## 5. CONCLUSION

Various equations relating magnetic constant and Bohr's radius for any atom and consequently effective nuclear charge were derived. Calculated results for Bohr's radius and effective nuclear charge of all elements (not just for hydrogen or hydrogenic atom) showed that magnetic constant like fine structure constant can be separately or jointly related to the radius of elements. Several pieces of evidence of this issue expressed via different equations may justify or validate Bohr's classical model. If the mathematical equations can reproduce the parameters, 'a' and  $Z_{eff}$  for hydrogen, multi-electron atoms could not be the exception.

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## COMPETING INTERESTS

Author has declared that no competing interests exist.

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