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An Efficient Method for Solving Linear and Nonlinear System of Partial Differential Equations

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Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

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Abstract

In this paper, Aboodh transform which is based on the Adomian decomposition method (AADM) is introduced for the approximate solution of the linear and nonlinear systems of partial differential equations. This method is very powerful and efficient techniques for solving different kinds of linear and nonlinear System of PDEs. The result reveals that the proposed method is very efficient, simple and can be applied to linear and nonlinear problems.

Keywords: Aboodh−Adomian decomposition method; systems of PDEs; Adomian polynomials.

1 Introduction

Systems of partial differential equations (PDEs) [1-4] arise in many scientific models such as the propagation of shallow water waves and in examining the chemical reaction –diffusion model of Brusselator. the general ideas and the essential features of these systems are of wide [5-6] applicability. In the recent

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years, many authors have devoted their attention to study solutions of nonlinear system ofpartial differential equations using various methods. Among these attempts are the homotopy perturbation method, variational iteration method [7–8], Laplace decomposition method method [9], differential transform method [10], Homotopy Perturbation Method [11-12], projected differential transform method, and Adomian decomposition method the Adomian decomposition approach has been applied to obtain formal solutions to a wide class of both deterministic and stochastic PDEs. The decomposition method has emerged as an alternative method for solving a wide range of problems whose mathematical models involve algebraic, differential, integral, integro- differential, higher-order ordinary differential equations,partial differential equations (PDEs) and systems [13-21]. These works are summarized in the following. In [13] used the Adomian's Technique for solving an Nonlinear Wave Equation. Hardik S. Patel and Ramakanta Meher. [14] used Modified Adomian decomposition method for solving eleventh-order initial and boundary value problems. Mohammed E. A. Rabie et al. [15] studied systems of linear partial differential equation by using Adomian and modified decomposition methods. Hardik S. Patel, Ramakanta Meher. [16] Presented an Application of modified Adomian decomposition method for solving higher order boundary value problem. Guellal et al. [17] used the decomposition method for solving differentia systems coming from physics. They gave a comparison Between the Runge-Kutta method and the decomposition technique. Trushit Patel and Ramakanta Meher [18] used the Adomian decomposition method for solving Infiltration Problem Arising in Farmland Drainage. In [19], the Adomian's scheme was used for solving differential systems for modeling the HIV immune dynamics. Mustafa Inc. [20] used the decomposition method for solving partial differential equations numerically. Hardik S. Patel, Ramakanta Meher [21] presented an Application of Laplace Adomian Decomposition Method for the soliton solutions of Boussinesq-Burger equations. Recently, Khalid Aboodh, has introduced a new integral transform, named the Aboodh transform [22-26], and it has further applied to the solution of ordinary and partial differential equations. In this article, we use AboodhAdomian Decomposition Method together to solve Nonlinear System of PDEs.

2 Aboodh transform

A new transform called the Aboodh transform defined for function of exponential order we consider functions in the set A**,** defined by:

$$A = \{f(t) : \exists\, M\,, k_1, k_2 > 0\,, |f(t)| < Me^{-vt}\} \tag{1}$$

For a given function in the set M must be finite number,k_1, k_2 may be finite or infinite. Aboodh transform which is defined by the integral equation

$$A[f(t)] = K(v) = \frac{1}{v}\int_0^\infty f(t)e^{-vt}\,dt \ \ , t \geq 0\,, \mathrm{k}_1 \leq v \leq \mathrm{k}_2 \tag{2}$$

2.1 Aboodh transform of some functions

$$A[1] = \frac{1}{v^2} \quad , \quad A[t^n] = \frac{n!}{v^{n+2}}$$

$$A[e^{at}] = \frac{1}{v^2 - av} \quad , \quad A[e^{-at}] = \frac{1}{v^2 + av}.$$

$$A[\sin(at)] = \frac{a}{v(v^2 + a^2)} \quad , \quad A[\cos(at)] = \frac{1}{(v^2 + a^2)}.$$

$$A[\sinh(at)] = \frac{a}{v(v^2 - a^2)} \quad , \quad A[\cosh(at)] = \frac{1}{(v^2 - a^2)}.$$

2.2 Aboodh transform of some partial derivative

$$A[u(x,t)] = K(x,v) \ \ , \quad A\left[\frac{\partial u(x,t)}{\partial x}\right] = K'(x,v)\ , \ A\left[\frac{\partial^2 u(x,t)}{\partial x^2}\right] = K''(x,v) \quad ,$$

$$A\left[\frac{\partial^n u(x,t)}{\partial x^n}\right] = K^{(n)}(x,v)\ , \quad A\left[\frac{\partial u(x,t)}{\partial t}\right] = v\,K(x,v) - \frac{u(x,0)}{v}$$

$$A\left[\frac{\partial^2 u(x,t)}{\partial t^2}\right] = v^2 K(x,v) - \frac{\frac{\partial u(x,0)}{\partial t}}{v} - u(x,0)$$

3 Aboodh−Adomian Decomposition Method

In this section, we present a Aboodh -Adomian decomposition method for solving of partial differential equations written in an operator form

$$\begin{cases} L_t U + R_1(U,V) + N_1(U,V) = G_1 \\ L_t V + R_2(U,V) + N_2(U,V) = G_2 \end{cases} \tag{3}$$

With the initial conditions

$$U(x,0) = f_1(x), V(x,0) = f_2(x) \tag{4}$$

Where L_t is considered a first-order partial differential operator, R_1, R_2 and N_1, N_2 are linear and nonlinear operators, respectively. And G_1, G_2 are source terms. The method consists of first applying the Aboodh transform to both sides of equations in system (3):

$$\begin{cases} \{A[L_t U] + A[R_1(U,V)] + A[N_1(U,V)] = A[G_1] \\ (A[L_t V] + A[R_2(U,V)] + A[N_2(U,V)] = A[G_2] \end{cases} \tag{5}$$

Using the differentiation property of Aboodh transform and above initial conditions (4), we have:

$$\begin{cases} E\{U(x,t)\} = \frac{f_1(x)}{v^2} + \frac{1}{v}(A[G_1] - A[R_1(U,V)] - A[N_1(U,V)]) \\ E\{V(x,t)\} = \frac{f_2(x)}{v^2} + \frac{1}{v}(A[G_2] - A[R_2(U,V)] - A[N_2(U,V)]) \end{cases} \tag{6}$$

Operating with the Aboodh inverse on both sides of Eq.(6) gives:

$$\begin{cases} U(x,t) = f_1(x) + A^{-1}\left[\frac{1}{v}(A[G_1] - A[R_1(U,V)] - A[N_1(U,V)])\right] \\ V(x,t) = f_2(x) + A^{-1}\left[\frac{1}{v}(A[G_2] - A[R_2(U,V)] - A[N_2(U,V)])\right] \end{cases} \tag{7}$$

The Adomain decomposition method defines the solutions $U(x,t)$ and $V(x,t)$ by an infinite series

$$U(x,t) = \sum_{n=0}^{\infty} U_n(x,t) \qquad , \qquad V(x) = \sum_{n=0}^{\infty} V_n(x,t) \tag{8}$$

Where the components $U_n(x,t)$ and $V_n(x,t)$ are usually determined recurrently. The nonlinear operators N_1, N_2 can be decomposed into an infinite series of polynomials given by

$$N_1 = \sum_{n=0}^{\infty} B_n \qquad , \qquad N_2 = \sum_{n=0}^{\infty} \tilde{B}_n \tag{9}$$

Where B_n and $\tilde{B}_n$ are the so called Adomain polynomials of $U_0, U_1, U_2, \dots U_n$ and $V_0, V_1, V_2, \dots V_n$ respectively. They are determined by the following relations:

$$B_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}[N_1(\lambda^i u_i)]_{\lambda=0}\ , \quad \tilde{B}_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}[N_2(\lambda^i u_i)]_{\lambda=0} \quad n = 0,1,2,\dots \tag{10}$$

Substituting (8) and (9) into (7), gives

$$\begin{cases} \sum_{n=0}^{\infty} U_n = f_1(x) + A^{-1}\left[\frac{1}{v}\left(A[G_1] - A[R_1(\sum_{n=0}^{\infty} U_n , \sum_{n=0}^{\infty} V_n)] - A[N_1(\sum_{n=0}^{\infty} A_n)]\right)\right] \\ \sum_{n=0}^{\infty} V_n = f_2(x) + A^{-1}\left[\frac{1}{v}\left(A[G_2] - A[R_2(\sum_{n=0}^{\infty} U_n , \sum_{n=0}^{\infty} V_n)] - A[N_2(\sum_{n=0}^{\infty} A_n)]\right)\right] \end{cases} \quad (11)$$

Applying the linearity of the Aboodh transform, we define the recursive relations are given by:

$$\begin{cases} U_0 = f_1(x) + A^{-1}\left[\frac{1}{v}(A[G_1])\right] \\ U_{n+1} = -A^{-1}\left[\frac{1}{v}\left(A[R_1(\sum_{n=0}^{\infty} U_n , \sum_{n=0}^{\infty} V_n)] + A[N_2(\sum_{n=0}^{\infty} A_n)]\right)\right] \end{cases} \quad (12)$$

and ,

$$\begin{cases} V_0 = f_2(x) + A^{-1}\left[\frac{1}{v}(A[G_2])\right] \\ V_{n+1} = -A^{-1}\left[\frac{1}{v}\left(A[R_1(\sum_{n=0}^{\infty} U_n , \sum_{n=0}^{\infty} V_n)] + A[N_1(\sum_{n=0}^{\infty} A_n)]\right)\right] \end{cases} \quad (13)$$

4 Applications

In this section, we use the AADM to solve homogeneous and inhomogeneous linear system of partial differential equations and homogeneous and inhomogeneous nonlinear system of partial differential equations.

Example 3.1

Consider the homogeneous linear system of PDEs:

$$\begin{cases} U_t - V_x + (U + V) = 0 \\ V_t - U_x + (U + V) = 0 \end{cases} \quad (14)$$

With the initial conditions

$$U(x, 0) = \sinh x , V(x, 0) = \cosh x \quad (15)$$

Taking the Aboodh transform on both sides of Eq. (14) then, by using the differentiation property of Aboodh transform and initial conditions (15) gives

$$\begin{cases} E\{U(x, t)\} = \frac{\sinh x}{v^2} + \frac{1}{v}(A[V_x - (U + V)]) \\ E\{V(x, t)\} = \frac{\cosh x}{v^2} + \frac{1}{v}(A[U_x - (U + V)]) \end{cases} \quad (16)$$

Operating with the Aboodh inverse on both sides of Eq. (16) gives:

$$\begin{cases} U(x, t) = \sinh x + A^{-1}\left[\frac{1}{v}(A[V_x - (U + V)])\right] \\ V(x, t) = \cosh x + A^{-1}\left[\frac{1}{v}(A[U_x - (U + V)])\right] \end{cases} \quad (17)$$

The AADM defines the solutions $U(x, t), V(x, t)$ by the series

$$U = \sum_{n=0}^{\infty} U_n \quad , \quad V = \sum_{n=0}^{\infty} V_n \quad (18)$$

and the terms U_x and V_x by an infinite series

$$U_x = \sum_{n=0}^{\infty} U_{n_x} \quad , \quad V_x = \sum_{n=0}^{\infty} V_{n_x} \quad (19)$$

Substituting series in (18) and (19) into both sides of Eq. (17) yields

$$\begin{cases}\sum_{n=0}^{\infty} U_n = \sinh x + A^{-1}\left[\frac{1}{v}\left(A\left[\sum_{n=0}^{\infty} V_{n_x} - \left(\sum_{n=0}^{\infty} U_n + \sum_{n=0}^{\infty} V_n\right)\right]\right)\right] \\ \sum_{n=0}^{\infty} V_n = \cosh x + A^{-1}\left[\frac{1}{v}\left(A\left[\sum_{n=0}^{\infty} U_{n_x} - \left(\sum_{n=0}^{\infty} U_n + \sum_{n=0}^{\infty} V_n\right)\right]\right)\right]\end{cases} \quad (20)$$

Now we define the following recursively formula:

$$\begin{cases}U_0 = \sinh x \\ V_0 = \cosh x\end{cases} \quad (21)$$

and ,

$$\begin{cases}U_{n+1} = A^{-1}\left[\frac{1}{v}\left(A\left[V_{n_x} - (U_n + V_n)\right]\right)\right] \\ U_{n+1} = A^{-1}\left[\frac{1}{v}\left(A\left[U_{n_x} - (U_n + V_n)\right]\right)\right]\end{cases} \quad , n \geq 0 \quad (22)$$

$$U_1 = A^{-1}\left[\frac{1}{v}\left(A\left[V_{0_x} - (U_0 + V_0)\right]\right)\right] = A^{-1}\left[\frac{1}{v}\left(A[\sinh x - (\sinh x + \cosh x)]\right)\right] = -t\cosh x$$
$$V_1 = A^{-1}\left[\frac{1}{v}\left(A\left[U_{0_x} - (U_0 + V_0)\right]\right)\right] = A^{-1}\left[\frac{1}{v}\left(A[\cosh x - (\sinh x + \cosh x)]\right)\right] = -t\sinh x$$
$$U_2 = A^{-1}\left[\frac{1}{v}\left(A\left[V_{1_x} - (U_1 + V_1)\right]\right)\right] = A^{-1}\left[\frac{1}{v}\left(A[-t\cosh x - (-t\sinh x - t\cosh x)]\right)\right] = \frac{t^2}{2}\sinh x$$
$$V_2 = A^{-1}\left[\frac{1}{v}\left(A\left[U_{1_x} - (U_1 + V_1)\right]\right)\right] = A^{-1}\left[\frac{1}{v}\left(A[-t\sinh x - (-t\cosh x - t\sinh x)]\right)\right] = \frac{t^2}{2}\cosh x$$

Proceeding in a similar way, we have:

$$U_3 = A^{-1}\left[\frac{1}{v}\left(A\left[V_{2_x} - (U_2 + V_2)\right]\right)\right] = -\frac{t^3}{3!}\cosh x$$

$$V_3 = A^{-1}\left[\frac{1}{v}\left(A\left[U_{2_x} - (U_2 + V_2)\right]\right)\right] = -\frac{t^3}{3!}\sinh x$$

And so on for other components. Using (8), the series solutions are therefore given by

$$\begin{cases}U = \sinh x\left(1 + \frac{t^2}{2!} + \frac{t^4}{4!} \ldots\right) - \cosh x\left(t + \frac{t^3}{3!} + \frac{t^5}{5!} \ldots\right) \\ V = \cosh x\left(1 + \frac{t^2}{2!} + \frac{t^4}{4!} \ldots\right) - \sinh x\left(t + \frac{t^3}{3!} + \frac{t^5}{5!} \ldots\right)\end{cases} \quad (23)$$

In series form, we can find the exact solutions

$$U = \sinh(x - t) \quad , \quad V = \cosh(x - t).$$

Example 3.2

Consider the inhomogeneous linear system of PDEs:

$$\begin{cases}U_t - V_x - (U - V) = -2 \\ V_t + U_x - (U - V) = -2\end{cases} \quad (24)$$

With the initial conditions

$$U(x,0) = 1 + e^{x} \quad , \quad V(x,0) = -1 + e^{x} \tag{25}$$

Taking the Aboodh transform on both sides of Eq. (24) then, by using the differentiation property of Aboodh transform and initial conditions (25) gives

$$\begin{cases} E\{U(x,t)\} = \frac{1+e^{x}}{v^2} + \frac{1}{v}(A[-2 + V_x + (U - V)]) \\ E\{V(x,t)\} = \frac{-1+e^{x}}{v^2} + \frac{1}{v}(A[-2 - U_x + (U - V)]) \end{cases} \tag{26}$$

Operating with the Aboodh inverse on both sides of Eq. (26) gives:

$$\begin{cases} U(x,t) = 1 + e^{x} - 2t + A^{-1}\left[\frac{1}{v}(A[V_x + (U - V)])\right] \\ V(x,t) = -1 + e^{x} - 2t + A^{-1}\left[\frac{1}{v}(A[-U_x + (U - V)])\right] \end{cases} \tag{27}$$

The LADM defines the solutions $U(x,t), V(x,t)$ by the series

$$U = \sum_{n=0}^{\infty} U_n \quad , \quad V = \sum_{n=0}^{\infty} V_n \tag{28}$$

and the terms U_x and V_x by an infinite series

$$U_x = \sum_{n=0}^{\infty} U_{n_x} \quad , \quad V_x = \sum_{n=0}^{\infty} V_{n_x} \tag{29}$$

Substituting series in (28) and (29) into both sides of Eq. (27) yields

$$\begin{cases} \sum_{n=0}^{\infty} U_n = 1 + e^{x} - 2t + A^{-1}\left[\frac{1}{v}\left(A\left[\sum_{n=0}^{\infty} V_{n_x} + \left(\sum_{n=0}^{\infty} U_n - \sum_{n=0}^{\infty} V_n\right)\right]\right)\right] \\ \sum_{n=0}^{\infty} V_n = -1 + e^{x} - 2t + A^{-1}\left[\frac{1}{v}\left(A\left[-\sum_{n=0}^{\infty} U_{n_x} + \left(\sum_{n=0}^{\infty} U_n - \sum_{n=0}^{\infty} V_n\right)\right]\right)\right] \end{cases} \tag{30}$$

Now we define the following recursively formula:

$$\begin{cases} U_0 = 1 + e^{x} - 2t \\ V_0 = -1 + e^{x} - 2t \end{cases} \tag{31}$$

and ,

$$\begin{cases} U_{n+1} = A^{-1}\left[\frac{1}{v}\left(A\left[V_{n_x} + (U_n - V_n)\right]\right)\right] \\ U_{n+1} = A^{-1}\left[\frac{1}{v}\left(A\left[-U_{n_x} + (U_n - V_n)\right]\right)\right] \end{cases} \quad , n \geq 0 \tag{32}$$

$$U_1 = A^{-1}\left[\frac{1}{v}\left(A\left[V_{0_x} + (U_0 - V_0)\right]\right)\right] = A^{-1}\left[\frac{1}{v}(A[e^{x} + 1 + e^{x} - 2t - (-1 + e^{x} - 2t))])\right] = t(e^{x} + 2)$$

$$V_1 = A^{-1}\left[\frac{1}{v}\left(A\left[-U_{0_x} + (U_0 - V_0)\right]\right)\right] = A^{-1}\left[\frac{1}{v}(A[-e^{x} + 1 + e^{x} - 2t - (-1 + e^{x} - 2t))])\right]$$
$$= t(-e^{x} + 2)$$

Proceeding in a similar way, we have:

$$U_2 = A^{-1}\left[\frac{1}{v}\left(A\left[V_{1_x} + (U_1 - V_1)\right]\right)\right] = \frac{t^{2}}{2!} e^{x}$$
$$V_2 = A^{-1}\left[\frac{1}{v}\left(A\left[-U_{1_x} + (U_1 - V_1)\right]\right)\right] = \frac{t^{2}}{2!} e^{x}$$

$$U_3 = A^{-1}\left[\frac{1}{v}\left(A\left[V_{2_x} + (U_2 - V_2)\right]\right)\right] = \frac{t^3}{3!}\, e^x$$

$$V_3 = A^{-1}\left[\frac{1}{v}\left(A\left[-U_{2_x} + (U_2 - V_2)\right]\right)\right] = -\frac{t^3}{3!}\, e^x$$

And so on for other components. Using (8), the series solutions are therefore given by

$$\begin{cases} U = 1 + e^x\left(1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + \cdots\right) \\ V = -1 + e^x\left(1 - t + \frac{t^2}{2!} - \frac{t^3}{3!} + \cdots\right) \end{cases} \tag{33}$$

In series form , we can find the exact solutions

$$U = 1 + e^{x+t} \quad , \quad V = -1 + e^{x-t}.$$

Example 3.2

Consider the inhomogeneous nonlinear system of PDEs:

$$\begin{cases} U_t + VU_x + U = 1 \\ V_t - UV_x - V = 1 \end{cases} \tag{34}$$

With the initial conditions

$$U(x,0) = e^x \quad , \quad V(x,0) = e^{-x} \tag{35}$$

Taking the Aboodh transform on both sides of Eq. (34) then, by using the differentiation property of Aboodh transform and initial conditions (35) gives

$$\begin{cases} \mathrm{E}\{\mathrm{U(x,t)}\} = \frac{e^x}{v^2} + \frac{1}{v}(A[1 - VU_x - U]) \\ \mathrm{E}\{\mathrm{V(x,t)}\} = \frac{e^{-x}}{v^2} + \frac{1}{v}(A[1 + UV_x + V]) \end{cases} \tag{36}$$

Operating with the Aboodh inverse on both sides of Eq. (36) gives:

$$\begin{cases} U(x,t) = e^x + t - A^{-1}\left[\frac{1}{v}(A[VU_x + U])\right] \\ V(x,t) = e^{-x} + t + A^{-1}\left[\frac{1}{v}(A[UV_x + V])\right] \end{cases} \tag{37}$$

We represent $U(x,t), V(x,t)$ by the infinite series (8) then, inserting these series into both sides of Eq. (37) yields

$$\begin{cases} \sum_{n=0}^{\infty} U_n = e^x + t - A^{-1}\left[\frac{1}{v}(A[\sum_{n=0}^{\infty} A_n + \sum_{n=0}^{\infty} U_n])\right] \\ \sum_{n=0}^{\infty} V_n = e^{-x} + t + A^{-1}\left[\frac{1}{v}\left(A\left[\sum_{n=0}^{\infty} \tilde{A}_n + \sum_{n=0}^{\infty} V_n\right]\right)\right] \end{cases} \tag{38}$$

Where A_n and $\tilde{A}_n$ are the so-called Adomian polynomials by (9) that represent the nonlinear terms VU_x and UV_x, respectively. We have a few terms of the Adomian polynomials for VU_x and UV_x, which are given by

$$A_0 = V_0 U_{0_x}$$

$$A_1 = V_1 U_{0_x} + V_0 U_{1_x}$$

$$A_2 = V_2 U_{0_x} + V_1 U_{1_x} + V_0 U_{2_x}$$
$$A_3 = V_3 U_{0_x} + V_2 U_{1_x} + V_1 U_{2_x} + V_0 U_{3_x}$$
$$\vdots$$

and,

$$A_0 = U_0 V_{0_x}$$
$$A_1 = U_1 V_{0_x} + U_0 V_{1_x}$$
$$A_2 = U_2 V_{0_x} + U_1 V_{1_x} + U_0 V_{2_x}$$
$$A_3 = U_3 V_{0_x} + U_2 V_{1_x} + U_1 V_{2_x} + U_0 V_{3_x}$$
$$\vdots$$

Now we define the following recursively formula:

$$\begin{cases} U_0 = t + e^x \\ V_0 = t + e^{-x} \end{cases} \tag{39}$$

and,

$$\begin{cases} U_{n+1} = -A^{-1}\left[\frac{1}{v}\left(A\left[\sum_{n=0}^{\infty} A_n + \sum_{n=0}^{\infty} U_n\right]\right)\right] \\ U_{n+1} = A^{-1}\left[\frac{1}{v}\left(A\left[\sum_{n=0}^{\infty} \tilde{A}_n + \sum_{n=0}^{\infty} V_n\right]\right)\right] \end{cases} \quad , n \geq 0 \tag{40}$$

$$U_1 = -t - \frac{t^2}{2!} - te^x - \frac{t^2}{2!} e^x$$

$$V_1 = -t - \frac{t^2}{2!} + te^{-x} - \frac{t^2}{2!} e^{-x}$$

$$U_2 = \frac{t^2}{2!} + t^2 e^x + \cdots$$

$$V_2 == \frac{t^2}{2!} + t^2 e^{-x} + \cdots$$

Similarly, we can find other components. Using (8), the series solutions are therefore given by

$$\begin{cases} U = e^x \left(1 - t + \frac{t^2}{2!} - \frac{t^3}{3!} + \cdots\right) \\ V = e^{-x} \left(1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + \cdots\right) \end{cases} \tag{41}$$

In series form, we can find the exact solutions

$$U = e^{x-t} \quad , \quad V = e^{-x+t}.$$

5 Conclusion

In this paper, Aboodh transform which is based on the Adomian decomposition method (AADM) is introduced for the approximate solution of the linear and nonlinear systems of partial differential equations. This methods is very powerful and efficient techniques for solving different kinds of linear and nonlinear System of PDEs. The method presents a useful way to develop an analytic treatment for these systems. The result reveals that the proposed method is very efficient, simple and can be applied to linear and nonlinear problems.

Competing Interests

Authors have declared that no competing interests exist.

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